13

NEWTON'S THEORY OF GRAVITY

Conceptual Questions

13.1. Newton's third law tells us that the forces are equal. They are also clearly equal when Newton's law of gravity is examined: $F_{12} = Gm_1m_2/r^2$ has the same value whether $m_1 = \text{Earth}$ and $m_2 = \text{sun}$ or vice versa.

13.2. The force of the star on planet 1 is $F_1 = G\frac{M_\text{s}m_1}{r_1^2}$. The force of the star on planet 2 is $F_2 = G\frac{M_\text{s}m_2}{r_2^2}$. Since $m_2 = 2m_1$, and $r_2 = 2r_1$,

$$\frac{F_2}{F_1} = \frac{G\frac{M_\text{s}(2m_1)}{(2r_1)^2}}{G\frac{M_\text{s}m_1}{r_1^2}} = \frac{2}{4} = \frac{1}{2}$$

13.3. **(a)** The force of the Earth on the first satellite is $F_1 = G\frac{M_\text{e}m_1}{r_1^2}$, while $F_2 = G\frac{M_\text{e}m_2}{r_2^2}$. Since $r_1 = r_2$, $\frac{F_1}{F_2} = \frac{m_1}{m_2} = \frac{1000\text{ kg}}{2000\text{ kg}} = \frac{1}{2}$.

(b) With $F_\text{net} = ma$, $\text{F}_1 = m_1a_1$ and $F_2 = m_2a_2$, so

$$\frac{a_1}{a_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{m_2}{m_1}\right) = \left(\frac{1}{2}\right)\left(\frac{2000\text{ kg}}{1000\text{ kg}}\right) = 1$$

The free-fall acceleration is the same for objects of different mass.

13.4. The astronauts can be "weightless" at any distance because an object is said to be weightless if it is in free fall (as in orbit). For the gravitational force to become zero, the spacecraft would have to be an infinite distance away.

13.5. The hammer will not fall to the Earth. The astronaut, shuttle, and hammer are all in free fall around the Earth (in an orbit), so the hammer has the same acceleration as the astronaut and does not move away from him.

13.6. The acceleration due to gravity is $g = G\frac{M}{R^2}$. With $g_1 = 20\text{ m/s}^2$, $M_2 = 2M_1$, and $R_2 = 2R_1$,

$$g_2 = G\frac{M_2}{R_2^2} = G\frac{(2M_1)}{(2R_1)^2} = G\frac{M_1}{R_1^2}\left(\frac{1}{2}\right) = \frac{1}{2}g_1 = 10\text{ m/s}^2$$

13.7. The gravitational potential energy is negative because we choose to place the zero point of potential energy at infinity $(U_{\text{at }\infty} = 0)$. With this choice, the gravitational potential energy is negative because the conservative force of gravity is attractive. The two masses will gain kinetic energy as they approach each other.

13.8. The escape speed from Planet X is

$$v_{\text{escape X}} = \sqrt{\frac{2GM_{\text{X}}}{R_{\text{X}}}} = 10{,}000 \text{ m/s}.$$

Planet Y is twice as dense as Planet X but the same size so has twice the mass. Therefore

$$v_{\text{escape Y}} = \sqrt{\frac{2G(2M_{\text{X}})}{R_{\text{X}}}} = \sqrt{2}v_{\text{escape X}} = 14{,}142 \text{ m/s}$$

13.9. The Earth's new orbital period would be about 11.9 years. For circular orbits, Kepler's third law relates orbital period to radius and can be written $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$. Here M is the mass of the body being orbited (the sun in this problem). The orbital period is independent of the mass of the orbiting body.

13.10. For a circular orbit, $v = \sqrt{GM/r}$ (Equation 13.22). As r decreases, v increases, so the satellite speeds up as the satellite spirals inward.

Exercises and Problems

Exercises

Section 13.3 Newton's Law of Gravity

13.1. Model: Model the sun (s) and the earth (e) as spherical masses. Due to the large difference between your size and mass and that of either the sun or the earth, a human body can be treated as a particle.

Solve: $F_{\text{s on you}} = \dfrac{GM_{\text{s}}M_{\text{y}}}{r_{\text{s-e}}^2}$ and $F_{\text{e on you}} = \dfrac{GM_{\text{e}}M_{\text{y}}}{r_{\text{e}}^2}$

Dividing these two equations gives

$$\frac{F_{\text{s on y}}}{F_{\text{e on y}}} = \left(\frac{M_{\text{s}}}{M_{\text{e}}}\right)\left(\frac{r_{\text{e}}}{r_{\text{s-e}}}\right)^2 = \left(\frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}\right)\left(\frac{6.37\times10^{6}\text{ m}}{1.50\times10^{11}\text{ m}}\right)^2 = 6.00\times10^{-4}$$

13.2. Model: Model the sun (s), the moon (m), and the earth (e) as spherical masses.

Solve: $F_{\text{s on m}} = \dfrac{GM_{\text{s}}M_{\text{m}}}{r_{\text{s-m}}^2}$ and $F_{\text{e on m}} = \dfrac{GM_{\text{e}}M_{\text{m}}}{r_{\text{e-m}}^2}$

Dividing the two equations and using the astronomical data from Table 13.2,

$$\frac{F_{\text{s on m}}}{F_{\text{e on m}}} = \left(\frac{M_{\text{s}}}{M_{\text{e}}}\right)\left(\frac{r_{\text{e-m}}}{r_{\text{s-m}}}\right)^2 = \left(\frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}\right)\left(\frac{3.84\times10^{8}\text{ m}}{1.50\times10^{11}\text{ m}}\right)^2 = 2.18$$

Note that the sun-moon distance is not noticeably different from the tabulated sun-earth distance.

13.3. Model: Assume the two lead balls are spherical masses.

Solve: (a) $F_{1\text{ on }2} = F_{2\text{ on }1} = \dfrac{Gm_1m_2}{r^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(10\text{ kg})(0.100\text{ kg})}{(0.10\text{ m})^2} = 6.7\times10^{-9}\text{ N}$

(b) The ratio of the above gravitational force to the gravitational force on the 100 g ball is

$$\frac{6.67\times10^{-9}\ \text{N}}{(0.100\ \text{kg})(9.8\ \text{m/s}^2)}=6.8\times10^{-9}$$

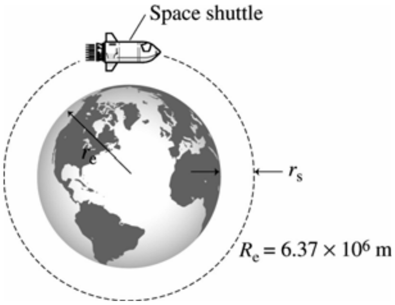
Assess: The answer in part (b) shows how small is the gravitational force between two lead balls separated by 10 cm compared to the gravitational force on the 100 g ball.

13.4. Model: Model the woman (w) and the man (m) as spherical masses or particles.

Solve: $F_{\text{w on m}}=F_{\text{m on w}}=\dfrac{GM_\text{w}M_\text{m}}{r_\text{m-w}^2}=\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(50\ \text{kg})(70\ \text{kg})}{(1.0\ \text{m})^2}=2.3\times10^{-7}\ \text{N}$

13.5. Model: Model the earth (e) as a sphere and apply Newton’s law of gravitation.

Visualize:



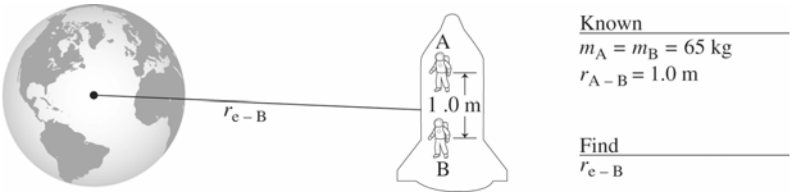
The space shuttle or a 1.0 kg sphere (s) in the space shuttle is $R_\text{e}+r_\text{s}=6.37\times10^6\ \text{m}+0.30\times10^6\ \text{m}=6.67\times10^6\ \text{m}$ away from the center of the earth.

Solve: $F_{\text{e on s}}=\dfrac{GM_\text{e}M_\text{s}}{(R_\text{e}+r_\text{s})^2}=\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(1.0\ \text{kg})}{(6.67\times10^6\ \text{m})^2}=9.0\ \text{N}$

Assess: This result is not much less than 9.8 N, which would be the force at the surface of the earth, because the altitude of the satellite is considerably less than the radius of the earth.

13.6. Model: Model the astronauts as particles so we can use Newton’s law of gravity.

Visualize:



Solve:

$$F_{\text{A on B}}=F_{\text{e on B}}$$

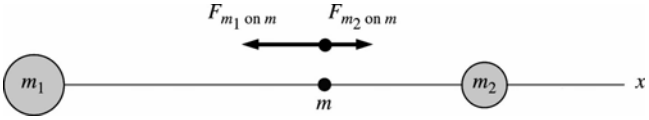
$$G\frac{m_\text{A}m_\text{B}}{r_{\text{A}-\text{B}}^2}=G\frac{m_\text{e}m_\text{B}}{r_{\text{e}-\text{B}}^2}$$

Cancel G and m_B and solve for $r_{\text{e}-\text{B}}$.

$$r_{\text{e}-\text{B}}=\sqrt{r_{\text{A}-\text{B}}^2\left(\frac{m_\text{e}}{m_\text{A}}\right)}=\sqrt{(1.0\ \text{m})^2\left(\frac{5.98\times10^{24}\ \text{kg}}{65\ \text{kg}}\right)}=3.0\times10^{11}\ \text{m}$$

Assess: This is twice the distance from the earth to the sun, or 2 au (astronomical units).

13.7. Visualize:



We placed the origin of the coordinate system on the 20 kg sphere (m_1). The sphere (m_2) with a mass of 10 kg is 20 cm away on the *x*-axis. The point at which the net gravitational force is zero must lie between the masses m_1 and m_2. This is because on such a point, the gravitational forces due to m_1 and m_2 are in opposite directions. As the gravitational force is directly proportional to the two masses and inversely proportional to the square of distance between them, the mass *m* must be closer to the 10-kg mass. The small mass *m*, if placed either to the left of m_1 or to the right of m_2, will experience gravitational forces from m_1 and m_2 pointing in the same direction, thus always leading to a nonzero force.

Solve:

$$F_{m_1 \text{ on } m} = F_{m_2 \text{ on } m} \quad \Rightarrow \quad G\frac{m_1 m}{x^2} = G\frac{m_2 m}{(0.20-x)^2} \Rightarrow \frac{20}{x^2} = \frac{10}{(0.20-x)^2}$$

$$10x^2 - 8x + 0.8 = 0 \quad \Rightarrow \quad x = 0.683 \text{ m and } 0.117 \text{ m}$$

The value $x = 68.3$ cm is unphysical in the current situation, since this point is not between m_1 and m_2. Thus, the mass should be placed 11.7 cm to the right of the larger mass. To two significant figures, this is 12 cm.

Section 13.4 Little g and Big G

13.8. Model: Model the sun (s) as a spherical mass.

Solve: (a) $g_{\text{sun surface}} = \dfrac{GM_\text{s}}{R_\text{s}^2} = \dfrac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30} \text{ kg})}{(6.96\times10^8 \text{ m})^2} = 274 \text{ m/s}^2$

(b) $g_{\text{sun at earth}} = \dfrac{GM_\text{s}}{r_\text{s-e}^2} = \dfrac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30} \text{ kg})}{(1.50\times10^{11} \text{ m})^2} = 5.90\times10^{-3} \text{ m/s}^2$

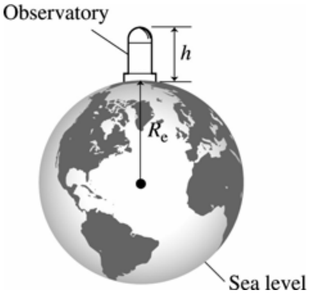
13.9. Model: Model the moon (m) and Jupiter (J) as spherical masses.

Solve: (a) $g_{\text{moon surface}} = \dfrac{GM_\text{m}}{R_\text{m}^2} = \dfrac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22} \text{ kg})}{(1.74\times10^6 \text{ m})^2} = 1.62 \text{ m/s}^2$

(b) $g_{\text{Jupiter surface}} = \dfrac{GM_\text{J}}{R_\text{J}^2} = \dfrac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27} \text{ kg})}{(6.99\times10^7 \text{ m})^2} = 25.9 \text{ m/s}^2$

13.10. Model: Model the earth (e) as a spherical mass.

Visualize: The acceleration due to gravity at sea level is 9.83 m/s^2 (see Table 13.1) and $R_\text{e} = 6.37\times10^6$ m (see Table 13.2).



Solve: $g_{\text{observatory}} = \dfrac{GM_\text{e}}{(R_\text{e}+h)^2} = \dfrac{GM_\text{e}}{R_\text{e}^2(1+h/R_\text{e})^2} = \dfrac{g_{\text{earth}}}{(1+h/R_\text{e})^2} = (9.83 - 0.0075)\ \text{m/s}^2$

Here $g_{\text{earth}} = GM_\text{e}/R_\text{e}^2$ is the acceleration due to gravity on a non-rotating earth, which is why we've used the value $9.83\ \text{m/s}^2$. Solving for *h*,

$$h = \left(\sqrt{\frac{9.83}{9.8225}} - 1\right)R_\text{e} = 2.4\ \text{km}$$

13.11. Solve:

$$g_{\text{surface}} = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.35\times10^{23}\ \text{kg})}{(2580\ \text{km})^2} = 1.35\ \text{m/s}^2$$

Assess: This moon has a much smaller mass than the earth, so we expected *g* to be smaller than 9.8 m/s^2.

13.12. Model: Assume the planet is spherical.

Visualize: We are given $R_\text{P} = 2R_\text{e},\ M_\text{P} = 5M_\text{e}$

Solve: Use ratios to minimize unnecessary calculation.

$$\frac{g_\text{P}}{g_\text{e}} = \frac{\dfrac{GM_\text{P}}{R_\text{P}^2}}{\dfrac{GM_\text{e}}{R_\text{e}^2}} = \left(\frac{R_\text{e}}{R_\text{P}}\right)^2 \frac{M_\text{P}}{M_\text{e}} = \left(\frac{R_\text{e}}{2R_\text{e}}\right)^2 \frac{5M_\text{e}}{M_\text{e}} = \frac{5}{4} \Rightarrow g_\text{P} = \tfrac{5}{4}g_\text{e} = \tfrac{5}{4}(9.8\ \text{m/s}^2) = 12\ \text{m/s}^2$$

Assess: The planet's larger mass more than offsets the larger radius, so g_P is a bit larger than earth's.

13.13. Model: Model the earth (e) as a spherical mass.

Solve: Let the acceleration due to gravity be $3.0g_{\text{surface}}$ when the earth is shrunk to a radius of *x*. Then,

$$g_{\text{surface}} = \frac{GM_\text{e}}{R_\text{e}^2} \text{ and } 3.0g_{\text{surface}} = \frac{GM_\text{e}}{x^2}$$

$$3\frac{GM_\text{e}}{R_\text{e}^2} = \frac{GM_\text{e}}{x^2} \quad\Rightarrow\quad x = \frac{R_\text{e}}{\sqrt{3.0}} = 0.58\,R_\text{e}$$

The earth's radius would need to be 0.58 times its present value.

13.14. Model: Model Planet Z as a spherical mass.

Solve: (a) $g_{\text{Z surface}} = \dfrac{GM_\text{Z}}{R_\text{Z}^2} \quad\Rightarrow\quad 8.0\ \text{m/s}^2 = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)M_\text{Z}}{(5.0\times10^6\ \text{m})^2} \quad\Rightarrow\quad M_\text{Z} = 3.0\times10^{24}\ \text{kg}$

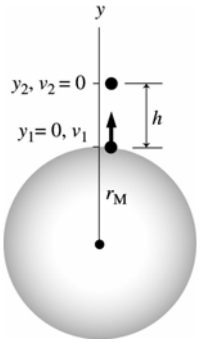
(b) Let *h* be the height above the north pole. Thus,

$$g_{\text{above N pole}} = \frac{GM_\text{Z}}{(R_\text{Z}+h)^2} = \frac{GM_\text{Z}}{R_\text{Z}^2(1+h/R_\text{Z})^2} = \frac{g_{\text{Z surface}}}{(1+h/R_\text{Z})^2} = \frac{8.0\ \text{m/s}^2}{\left(1+\dfrac{10.0\times10^6\ \text{m}}{5.0\times10^6\ \text{m}}\right)^2} = 0.89\ \text{m/s}^2$$

Section 13.5 Gravitational Potential Energy

13.15. Model: Model Mars (M) as a spherical mass. Ignore air resistance. Also consider Mars and the ball as an isolated system, so mechanical energy is conserved.

Visualize:



Solve: A height of 15 m is very small in comparison with the radius of earth or Mars. We can use flat-earth gravitational potential energy to find the speed with which the astronaut can throw the ball. On earth, with $y_i = 0$ m and $v_f = 0$ m/s, energy conservation gives

$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i \quad \Rightarrow \quad v_i = \sqrt{2g_e y_f} = \sqrt{2(9.81\ \text{m/s}^2)(15\ \text{m})} = 17.2\ \text{m/s}$$

Energy is also conserved on Mars, but the acceleration due to gravity is different.

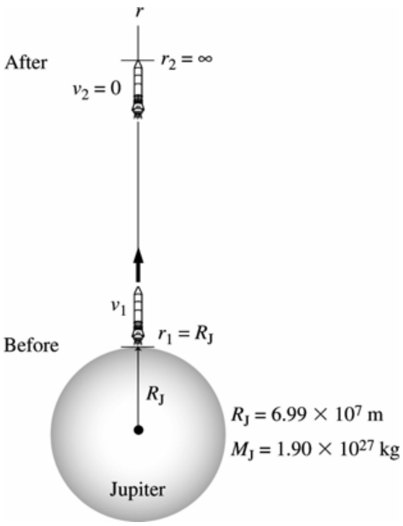
$$g_{\text{Mar's surface}} = \frac{GM_M}{R_M^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})}{(3.37\times10^6\ \text{m})^2} = 3.77\ \text{m/s}^2$$

On Mars, with $y_i = 0$ m and $v_f = 0$ m/s, energy conservation is

$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i \quad \Rightarrow \quad y_f = \frac{v_i^2}{2g_m} = \frac{(17.2\ \text{m/s})^2}{2(3.77\ \text{m/s}^2)} = 39\ \text{m}$$

13.16. Model: Model Jupiter as a spherical mass and the object as a point particle. The object and Jupiter form an isolated system, so mechanical energy is conserved. The minimum launch speed for escape, which is called the escape speed, allows an object to escape to an infinite distance from Jupiter (or, in general, from its partner object).

Visualize:



Solve: The energy conservation equation $K_2 + U_2 = K_1 + U_1$ is

$$\tfrac{1}{2}m_\text{o}v_2^2 - \frac{GM_\text{J}m_\text{o}}{r_2} = \tfrac{1}{2}m_\text{o}v_1^2 - \frac{GM_\text{J}m_\text{o}}{R_\text{J}}$$

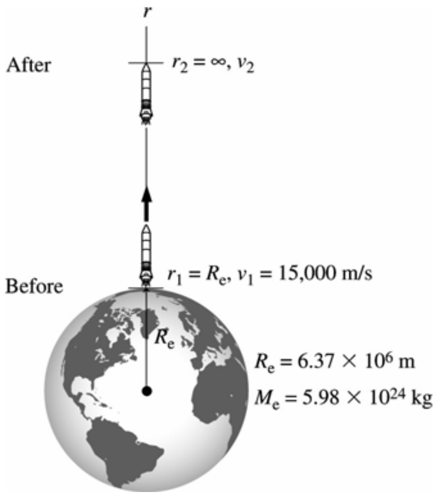
where R_J and M_J are the radius and mass of Jupiter. Using the asymptotic condition $v_2 = 0$ m/s as $r_2 \to \infty$,

$$0\text{ J} = \tfrac{1}{2}m_\text{o}v_1^2 - \frac{GM_\text{J}m_\text{o}}{R_\text{J}} \quad\Rightarrow\quad v_1 = \sqrt{\frac{2GM_\text{J}}{R_\text{J}}} = \sqrt{\frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27}\text{ kg})}{6.99\times10^{7}\text{ m}}} = 6.02\times10^4\text{ m/s}$$

Thus, the escape velocity from Jupiter is 60.2 km/s.

13.17. Model: Model the earth (e) as a spherical mass. Compared to the earth's size and mass, the rocket (r) is modeled as a particle. This is an isolated system, so mechanical energy is conserved.

Visualize:


r
After
$r_2 = \infty$, v_2
Before
$r_1 = R_e$, v_1 = 15,000 m/s
R_e
$R_e = 6.37 \times 10^6$ m
$M_e = 5.98 \times 10^{24}$ kg

Solve: The energy conservation equation $K_2 + U_2 = K_1 + U_1$ is

$$\tfrac{1}{2}m_\text{r}v_2^2 - \frac{GM_\text{e}m_\text{r}}{r_2} = \tfrac{1}{2}m_\text{r}v_1^2 - \frac{GM_\text{e}m_\text{r}}{R_\text{e}}$$

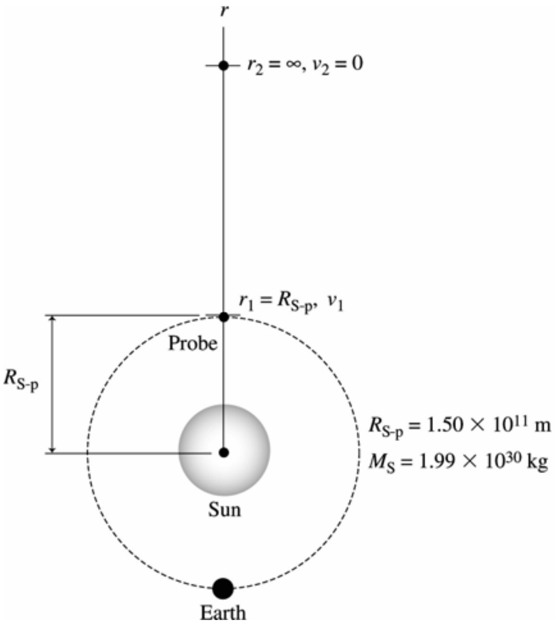
In the present case, $r_2 \to \infty$, so

$$\tfrac{1}{2}m_\text{r}v_2^2 = \tfrac{1}{2}m_\text{r}v_1^2 - \frac{GM_\text{e}m_\text{r}}{R_\text{e}} \quad\Rightarrow\quad v_2^2 = v_1^2 - \frac{2GM_\text{e}}{R_\text{e}}$$

$$v_2 = \sqrt{(1.5\times10^4\text{ m/s})^2 - \frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{6.37\times10^6\text{ m}}} = 10\text{ km/s}$$

13.18. Model: The probe and the sun form an isolated system, so mechanical energy is conserved. The minimum launch speed for escape, which is called the escape speed, allows an object to escape to an infinite distance from the sun, where the object will have slowed to zero speed with respect to the sun.

Visualize:



r
$r_2 = \infty$, $v_2 = 0$
$r_1 = R_{\text{S-p}}$, v_1
Probe
$R_{\text{S-p}}$
$R_{\text{S-p}} = 1.50 \times 10^{11}$ m
$M_S = 1.99 \times 10^{30}$ kg
Sun
Earth

We denote by m_p the mass of the probe. M_S is the sun's mass, and $R_{\text{S-p}}$ is the separation between the centers of the sun and the probe.

Solve: The conservation of energy equation $K_2 + U_2 = K_1 + U_1$ is

$$\frac{1}{2}m_p v_2^2 - \frac{GM_S m_p}{r_2} = \frac{1}{2}m_p v_1^2 - \frac{GM_S m_p}{R_{\text{S-p}}}$$

Using the condition $v_2 = 0$ m/s asymptotically as $r_2 \to \infty$,

$$\frac{1}{2}m_p v_{\text{escape}}^2 = \frac{GM_S m_p}{R_{\text{S-p}}} \quad \Rightarrow \quad v_{\text{escape}} = \sqrt{\frac{2GM_S}{R_{\text{S-p}}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{1.50\times10^{11}\ \text{m}}} = 4.21\times10^4\ \text{m/s}$$

13.19. Model: Assume the earth is spherical and the space elevator launching pad is 36,000 km from the center of the earth.

Visualize: See Example 13.2. In addition to R_e, the radius of the earth, we use *h*, the height above the earth.

Solve: The escape speed is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R_e + h}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(0.637+3.6)\times10^7\ \text{m}}} = 4.3\ \text{km/s}$$

Assess: This escape speed is less than half of what it is from the surface of the earth.

13.20. Model: Black holes are traditionally thought of having zero volume, but the "size" of a black hole is often given as the Schwarzschild radius, the radius of the event horizon.

Solve:

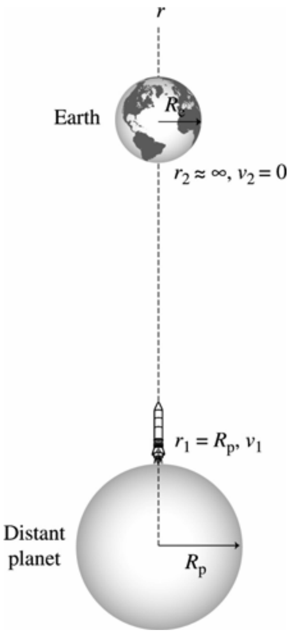
$$R_{\text{Sch}} = \frac{2GM}{v_{\text{escape}}^2} = \frac{2GM}{c^2} = \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)5(1.99\times10^{30}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2} = 14.7\ \text{km}$$

Assess: The Schwarzschild radius is proportional to the mass of the black hole and everything else is constant.

$$\frac{1}{2}m_p v_{\text{escape}}^2 = \frac{GM_s m_p}{R_{\text{s-p}}} \quad \Rightarrow \quad v_{\text{escape}} = \sqrt{\frac{2GM_s}{R_{\text{s-p}}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{1.50\times10^{11}\ \text{m}}} = 4.21\times10^4\ \text{m/s}$$

13.21. Model: Model the distant planet (p) and the earth (e) as spherical masses. Because both are isolated, the mechanical energy of the object on both the planet and the earth is conserved.

Visualize:



Let us denote the mass of the planet by M_p and that of the earth by M_e. Your mass is m_0. Also, acceleration due to gravity on the surface of the planet is g_p and on the surface of the earth is g_e. R_p and R_e are the radii of the planet and the earth, respectively.

Solve: (a) We are given that $M_\text{P} = 2M_\text{e}$ and $g_\text{p} = \frac{1}{4}g_\text{e}$.

Since $g_\text{p} = \dfrac{GM_\text{p}}{R_\text{p}^2}$ and $g_\text{e} = \dfrac{GM_\text{e}}{R_\text{e}^2}$, we have

$$\frac{GM_\text{p}}{R_\text{p}^2} = \frac{1}{4}\frac{GM_\text{e}}{R_\text{e}^2} = \frac{1}{4}\frac{G(M_\text{p}/2)}{R_\text{e}^2} \quad \Rightarrow \quad R_\text{p} = \sqrt{8}R_\text{e} = \sqrt{8}(6.37\times10^6\text{ m}) = 1.80\times10^7\text{ m}$$

(b) The conservation of energy equation $K_2 + U_2 = K_1 + U_1$ is

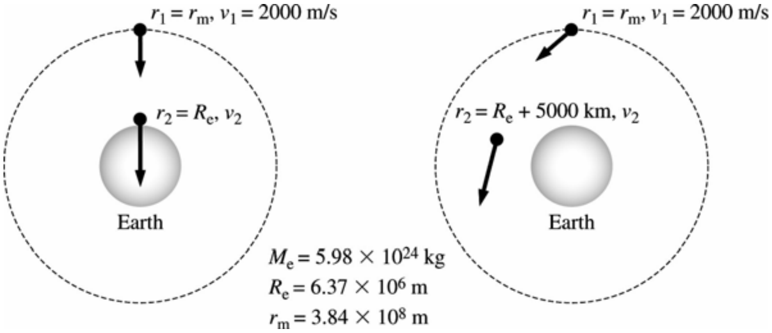
$$\frac{1}{2}m_0v_2^2 - \frac{GM_\text{p}m_0}{r_2} = \frac{1}{2}m_0v_1^2 - \frac{GM_\text{p}m_0}{R_\text{p}}$$

Using $v_2 = 0$ m/s as $r_2 \to \infty$, we have

$$\frac{1}{2}m_0v_\text{escape}^2 = \frac{GM_\text{p}m_0}{R_\text{p}}$$

$$v_\text{escape} = \sqrt{\frac{2GM_\text{p}}{R_\text{p}}} = \sqrt{\frac{2G(2M_\text{e})}{R_\text{p}}} = \sqrt{\frac{4(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{1.80\times10^7\text{ m}}} = 9.41\text{ km/s}$$

13.22. Model: Model the earth as a spherical mass and the meteoroids as point masses.
Visualize:


$r_1 = r_m$, $v_1 = 2000$ m/s
$r_2 = R_e$, v_2
Earth
$M_e = 5.98 \times 10^{24}$ kg
$R_e = 6.37 \times 10^6$ m
$r_m = 3.84 \times 10^8$ m
$r_1 = r_m$, $v_1 = 2000$ m/s
$r_2 = R_e + 5000$ km, v_2
Earth

Solve: (a) The energy conservation equation $K_2 + U_2 = K_1 + U_1$ gives

$$\tfrac{1}{2}mv_2^2 - \frac{GM_e m}{R_e} = \tfrac{1}{2}mv_1^2 - \frac{GM_e m}{r_m} \quad \Rightarrow \quad v_2 = \left[v_1^2 + 2GM_e\left(\frac{1}{R_e} - \frac{1}{r_m}\right)\right]^{1/2}$$

$$\left[(2000\ \text{m/s})^2 + 2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{3.85\times10^8\ \text{m}}\right)\right]^{1/2}$$

$$= 1.1\times10^4\ \text{m/s} = 11\ \text{km/s}$$

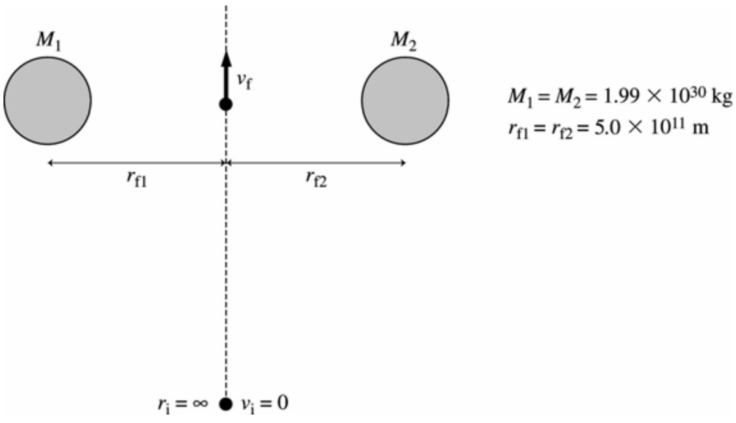
The speed does not depend on the meteoroid's mass.

(b) This part differs in that $r_2 = R_e + 5000\ \text{km} = 1.137\times10^7$ m. The shape of the meteoroid's trajectory is not important for using energy conservation. Thus

$$\tfrac{1}{2}mv_2^2 - \frac{GM_e m}{R_e + 5000\ \text{km}} = \tfrac{1}{2}mv_1^2 - \frac{GM_e m}{r_m} \quad \Rightarrow \quad v_2 = \left[v_1^2 + 2GM_e\left(\frac{1}{R_e + 5000\ \text{km}} - \frac{1}{r_m}\right)\right]^{1/2}$$

$$= 8.9\times10^3\ \text{m/s} = 8.9\ \text{km/s}$$

13.23. Model: Model the two stars as spherical masses, and the comet as a point mass. This is an isolated system, so mechanical energy is conserved.
Visualize:


M_1
M_2
v_f
$M_1 = M_2 = 1.99 \times 10^{30}$ kg
$r_{f1} = r_{f2} = 5.0 \times 10^{11}$ m
r_{f1}
r_{f2}
$r_i = \infty$
$v_i = 0$

In the initial state, the comet is far away from the two stars and thus it has neither kinetic energy nor potential energy. In the final state, as the comet passes through the midpoint connecting the two stars, it possesses both kinetic energy and potential energy.

Solve: The conversation of energy equation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ gives

$$\frac{1}{2}mv_\text{f}^2 - \frac{GMm}{r_{\text{f}1}} - \frac{GMm}{r_{\text{f}2}} = 0\text{ J} + 0\text{ J}$$

$$v_\text{f} = \sqrt{\frac{4GM}{r_\text{f}}} = \sqrt{\frac{4(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{0.50\times10^{12}\text{ m}}} = 32{,}600\text{ m/s} = 33\text{ km/s}$$

Assess: Note that the final velocity of 33 km/s does not depend on the mass of the comet.

Section 13.6 Satellite Orbits and Energies

13.24. Model: Model the sun (s) as a spherical mass and the asteroid (a) as a point particle.

Visualize: The asteroid, having mass m_a and velocity v_a, orbits the sun in a circle of radius r_a. The asteroid's time period is $T_\text{a} = 5.0$ earth years $= 1.58\times10^8$ s.

Solve: The gravitational force between the sun $(\text{mass} = M_\text{s})$ and the asteroid provides the centripetal acceleration required for circular motion.

$$\frac{GM_\text{s}m_\text{a}}{r_\text{a}^2} = \frac{m_\text{a}v_\text{a}^2}{r_\text{a}} \Rightarrow \frac{GM_\text{s}}{r_\text{a}} = \left(\frac{2\pi r_\text{a}}{T_\text{a}}\right)^2 \Rightarrow r_\text{a} = \left(\frac{GM_\text{s}T_\text{a}^2}{4\pi^2}\right)^{1/3}$$

Substituting $G = 6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2$, $M_\text{s} = 1.99\times10^{30}$ kg, and the time period of the asteroid, we obtain $r_\text{a} = 4.4\times10^{11}$ m. The velocity of the asteroid in its orbit will therefore be

$$v_\text{a} = \frac{2\pi r_\text{a}}{T_\text{a}} = \frac{2\pi(4.4\times10^{11}\text{ m})}{1.58\times10^8\text{ s}} = 1.7\times10^4\text{ m/s}$$

Solve: We give the answer to two significant figures because the asteroid period is given to two significant figures.

13.25. Model: Model the star (s) and the planet (p) as spherical masses.

Solve: From the planet's acceleration due to gravity, we find its mass to be

$$g_\text{p} = \frac{GM_\text{p}}{R_\text{p}^2}$$

$$M_\text{p} = \frac{g_\text{p}R_\text{p}^2}{G} = \frac{(12.2\text{ m/s}^2)(9.0\times10^6\text{ m})^2}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 1.5\times10^{25}\text{ kg}$$

(b) From the planet's orbital period, we find the mass of the star Omega to be

$$T^2 = \left(\frac{4\pi^2}{GM_\text{s}}\right)r^3$$

$$M_\text{s} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(2.2\times10^{11}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(402\times24\times3600\text{ s})^2} = 5.2\times10^{30}\text{ kg}$$

13.26. Model: Model the planet and satellites as spherical masses.

Visualize: Please refer to Figure EX13.26.

Solve: (a) The period of a satellite in a circular orbit is $T = [4\pi^2 r^3/(GM)]^{1/2}$ This is independent of the satellite's mass, so we can find the ratio of the periods of two satellites a and b:

$$\frac{T_\text{a}}{T_\text{b}} = \sqrt{\left(\frac{r_\text{a}}{r_\text{b}}\right)^3}$$

Satellite 2 has $r_2 = r_1$, so $T_2 = T_1 = 250$ min. Satellite 3 has $r_3 = (3/2)r_1$, so $T_3 = (3/2)^{3/2}T_1 = 459$ min.

(b) The force on a satellite is $F = GMm/r^2$. Thus the ratio of the forces on the two satellites a and b is

$$\frac{F_a}{F_b} = \left(\frac{r_b}{r_a}\right)^2 \left(\frac{m_a}{m_b}\right)$$

Satellite 2 has $r_2 = r_1$ and $m_2 = 2m_1$, so $F_2 = (1)^2(2)F_1 = 20{,}000\text{ N}$. Similarly, satellite 3 has $r_3 = (3/2)r_1$ and $m_3 = m_1$, so $F_3 = (2/3)^2(1)F_1 = 4440\text{ N}$.

(c) The speed of a satellite in a circular orbit is $v = (GM/r)^2$, so its kinetic energy is $K = \frac{1}{2}mv^2 = GMm/2r$. Thus the ratio of the kinetic energy of two satellites a and b is

$$\frac{K_a}{K_b} = \left(\frac{r_b}{r_a}\right)\left(\frac{m_a}{m_b}\right)$$

Satellite 3 has $r_3 = (3/2)r_1$ and $m_3 = m_1$, so $K_1/K_3 = (3/2)(1/1) = 3/2 = 1.50$.

13.27. Model: Model the sun (S) as a spherical mass and the satellite (s) as a point particle.
Visualize: The satellite, having mass m_s and velocity v_s, orbits the sun with a mass M_S in a circle of radius r_s.
Solve: The gravitational force between the sun and the satellite provides the necessary centripetal acceleration for circular motion. Newton's second law gives

$$\frac{GM_S m_s}{r_s^2} = \frac{m_s v_s^2}{r_s}$$

Because $v_s = 2\pi r_s/T_s$ where T_s is the period of the satellite, this equation simplifies to

$$\frac{GM_S}{r_s^2} = \frac{(2\pi r_s)^2}{T_s^2 r_s} \Rightarrow r_s^3 = \frac{GM_S T_s^2}{4\pi^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(24\times3600\text{ s})^2}{4\pi^2} \Rightarrow r_s = 2.9\times10^9\text{ m}$$

13.28. Model: Use Kepler's third law.
Visualize: Use subscripts P for planet, e for earth, V for Vega, and S for sun. Use *r* for the radius of an orbit.
Solve:

$$T^2 = \frac{4\pi^2}{GM}r^3 \Rightarrow r = \sqrt[3]{\frac{GM}{4\pi^2}T^2}$$

$$\frac{r_P}{r_e} = \frac{\sqrt[3]{\frac{GM_V}{4\pi^2}T_P^2}}{\sqrt[3]{\frac{GM_S}{4\pi^2}T_e^2}} = \sqrt[3]{\frac{M_V}{M_S}\frac{T_P^2}{T_e^2}} = \sqrt[3]{\frac{2.1M_S}{M_S}\left(\frac{55T_e}{T_e}\right)^2} = \sqrt[3]{2.1(55)^2} = 19$$

The radius of the planet's orbit is 19 times the radius of the earth's orbit, or 19 au (astronomical units).
Assess: The answer is about the same distance as Uranus is from our sun, so this is reasonable.

13.29. Model: Use Kepler's third law.
Visualize: Let *M* be the mass of the planet.
Solve:

$$T^2 = \frac{4\pi^2}{GM}r^3 \Rightarrow M = \frac{4\pi^2}{G}\frac{r^3}{T^2}$$

Do a side calculation: $\text{dist} = \text{speed}\times\text{time} \Rightarrow 2\pi r = vT \Rightarrow r = vT/2\pi$.

$$M = \frac{4\pi^2}{G}\frac{r^3}{T^2} = \frac{4\pi^2}{G}\frac{\left(\frac{vT}{2\pi}\right)^3}{T^2} = \frac{1}{2\pi G}v^3 T$$

$$= \frac{1}{2\pi(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}(7.5\text{ km/s})^3(28\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 1.0\times10^{26}\text{ kg}$$

Assess: This is a large Neptune-size planet.

13.30. Model: Model the earth (e) as a spherical mass and the satellite (s) as a point particle.

Visualize: The satellite has a mass is m_s and orbits the earth with a velocity v_s. The radius of the circular orbit is denoted by r_s and the mass of the earth by M_e.

Solve: The satellite experiences a gravitational force that provides the centripetal acceleration required for circular motion:

$$\frac{GM_\text{e}m_\text{s}}{r_\text{s}^2}=\frac{m_\text{s}v_\text{s}^2}{r_\text{s}} \Rightarrow r_\text{s}=\frac{GM_\text{e}}{v_\text{s}^2}=\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(5500\ \text{m/s})^2}=1.32\times10^7\ \text{m}$$

$$T_\text{s}=\frac{2\pi R_\text{s}}{v_\text{s}}=\frac{2\pi(1.32\times10^7\ \text{m})}{5500\ \text{m/s}}=1.51\times10^4\ \text{s} = 4.2\ \text{h}$$

13.31. Model: Model Mars (m) as a spherical mass and the satellite (s) as a point particle.

Visualize: The geosynchronous satellite whose mass is m_s and velocity is v_s orbits in a circle of radius r_s around Mars. Let us denote mass of Mars by M_m.

Solve: The gravitational force between the satellite and Mars provides the centripetal acceleration needed for circular motion:

$$\frac{GM_\text{m}m_\text{s}}{r_\text{s}^2}=\frac{m_\text{s}v_\text{s}^2}{r_\text{s}}=\frac{m_\text{s}(2\pi r_\text{s})^2}{r_\text{s}T_s^2} \Rightarrow r_\text{s}=\left(\frac{GM_\text{m}T_\text{s}^2}{4\pi^2}\right)^{1/3}$$

Using $G=6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, $M_m=6.42\times10^{23}$ kg, and $T_\text{s}=(24.8\ \text{hrs})=(24.8)(3600)\ \text{s}=89{,}280\ \text{s}$, we obtain $r_\text{s}=2.05\times10^7$ m. Thus, $\text{altitude}=r_\text{s}-R_\text{m}=1.72\times10^7$ m. The speed is the orbital circumference divided by the period, or

$$v_\text{s}=\frac{2\pi r_\text{s}}{T_\text{s}}=\frac{2\pi(2.05\times10^7\ \text{m/s})}{89{,}280\ \text{s}}=1.44\ \text{km/s}$$

13.32. Model: Model the earth (e) as a spherical mass and the satellite (s) as a point particle.

Visualize: Let *h* be the height from the surface of the earth where the acceleration due to gravity (g_altitude) is 10% of the surface value (g_surface).

Solve: (a) Since $g_\text{altitude}=(0.10)g_\text{surface}$, we have

$$\frac{GM_\text{e}}{(R_\text{e}+h)^2}=(0.10)\frac{GM_\text{e}}{R_\text{e}^2} \Rightarrow (R_\text{e}+h)^2=10R_\text{e}^2$$

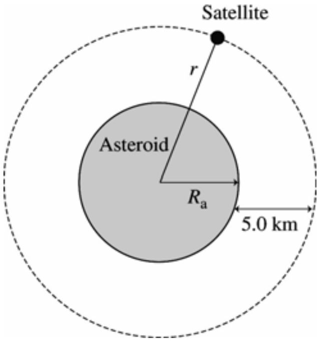
$$h=2.162R_\text{e} \Rightarrow h=(2.162)(6.37\times10^6\ \text{m})=1.377\times10^7\ \text{m}= 1.4\times10^7\ \text{m}$$

(b) For a satellite orbiting the earth at a height *h* above the surface of the earth, the gravitational force between the earth and the satellite provides the centripetal acceleration necessary for circular motion. For a satellite orbiting with velocity v_s,

$$\frac{GM_\text{e}m_\text{s}}{(R_\text{e}+h)^2}=\frac{m_\text{s}v_\text{s}^2}{(R_\text{e}+h)} \Rightarrow v_\text{s}=\sqrt{\frac{GM_\text{e}}{R_\text{e}+h}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(6.37\times10^6\ \text{m}+1.377\times10^7\ \text{m})}}=4.5\ \text{km/s}$$

13.33. Model: Assume a spherical asteroid and a point mass model for the satellite. This is an isolated system, so mechanical energy is conserved.

Visualize:



The orbital radius of the satellite is

$$r = R_a + h = 8{,}800 \text{ m} + 5{,}000 \text{ m} = 13{,}800 \text{ m}$$

Solve: (a) The speed of a satellite in a circular orbit is

$$v = \sqrt{\frac{GM}{r}} = \left[\frac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{16} \text{ kg})}{13{,}800 \text{ m}}\right]^{1/2} = 7.0 \text{ m/s}$$

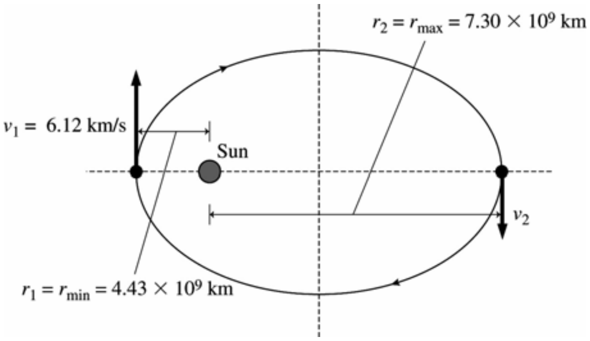
(b) The minimum launch speed for escape (v_i) will cause the satellite to stop asymptotically $(v_f = 0 \text{ m/s})$ as $r_f \to \infty$. Using the energy conservation equation $K_2 + U_2 = K_1 + U_1$, we get

$$\frac{1}{2}m_s v_f^2 - \frac{GM_a m_s}{r_f} = \frac{1}{2}m_s v_i^2 - \frac{GM_a m_s}{R_a} \quad \Rightarrow \quad 0 \text{ J} - 0 \text{ J} = \frac{1}{2}v_{\text{escape}}^2 - \frac{GM_a}{R_a}$$

$$v_{\text{escape}} = \sqrt{\frac{2GM_a}{R_a}} = \sqrt{\frac{2(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{16} \text{ kg})}{8800 \text{ m}}} = 12 \text{ m/s}$$

13.34. Model: Angular momentum is conserved for a particle following a trajectory of any shape.

Visualize:


$r_2 = r_{max} = 7.30 \times 10^9$ km
$v_1 = 6.12$ km/s
Sun
v_2
$r_1 = r_{min} = 4.43 \times 10^9$ km

For a particle in an elliptical orbit, the particle's angular momentum is $L = mrv_t = mrv\sin\beta$, where v is the velocity tangent to the trajectory and β is the angle between $\vec{r}$ and $\vec{v}$.

Solve: At the distance of closest approach (r_{min}) and also at the most distant point, $\beta = 90°$. Since there is no tangential force (the only force being the radial force) there is no torque, so angular momentum must be conserved:

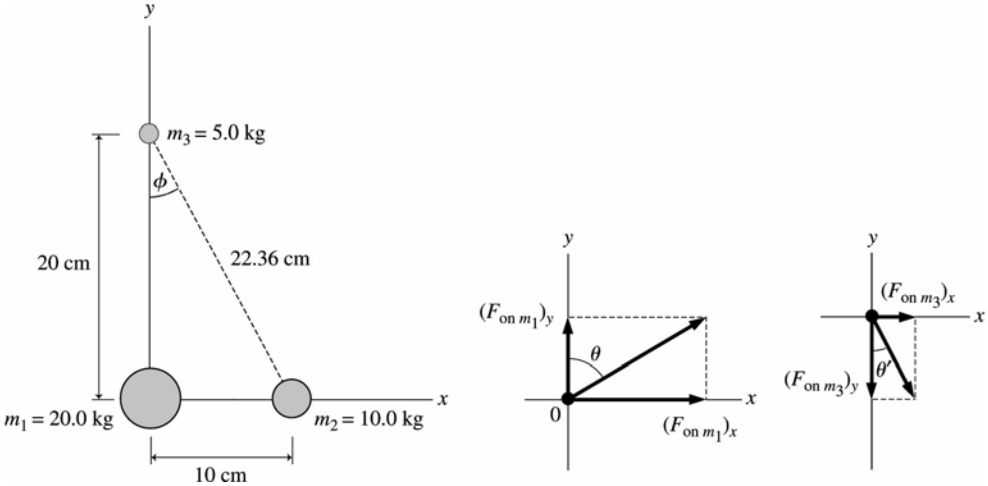
$$m_{\text{Pluto}} v_1 r_{\text{min}} = m_{\text{Pluto}} v_2 r_{\text{max}}$$

$$v_2 = v_1(r_{\text{min}}/r_{\text{max}}) = (6.12\times10^3 \text{ m/s})(4.43\times10^{12} \text{ m})/(7.30\times10^{12} \text{ m}) = 3.71 \text{ km/s}$$

Problems

13.35. Model: Treat the masses as point masses and use Newton's theory of gravitation. Use vector components to find the magnitude and direction of the net force.

Visualize: We placed the origin of the coordinate system on the 20.0 kg mass (m_1) so that the 5.0 kg mass (m_3) is on the y-axis and the 10.0 kg mass (m_2) is on the x-axis.



Solve: (a) The forces acting on the 20.0 kg mass (m_1) are

$$\vec{F}_{m_2 \text{ on } m_1} = \frac{gm_1m_2}{r_{12}^2}\hat{i} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(20.0\ \text{kg})(10.0\ \text{kg})}{(0.10\ \text{m})^2}\hat{i} = 1.33\times10^{-6}\hat{i}\ \text{N}$$

$$\vec{F}_{m_3 \text{ on } m_1} = \frac{gm_1m_3}{r_{13}^2}\hat{j} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(20.0\ \text{kg})(5.0\ \text{kg})}{(0.20\ \text{m})^2}\hat{j} = 1.67\times10^{-7}\hat{j}\ \text{N}$$

$$\vec{F}_{\text{on } m_1} = \vec{F}_{m_2 \text{ on } m_1} + \vec{F}_{m_3 \text{ on } m_1} = 1.33\times10^{-6}\hat{i}\ \text{N} + 1.67\times10^{-7}\hat{j}\ \text{N} \Rightarrow F_{\text{on } m_1} = 1.34\times10^{-6}\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{(\vec{F}_{\text{on } m_1})_x}{(\vec{F}_{\text{on } m_1})_y}\right) = \tan^{-1}\left(\frac{1.33\times10^{-6}\ \text{N}}{1.67\times10^{-7}\ \text{N}}\right) = 83°$$

Thus the force is $\vec{F}_{\text{on } m_1} = 1.3\times10^{-6}$ N, 83° cw from the $+y$-axis

(b) The forces acting on the 5.0 kg mass (m_3) are

$$\vec{F}_{m_1 \text{ on } m_3} = -\vec{F}_{m_3 \text{ on } m_1} = -1.67\times10^{-7}\hat{j}\ \text{N}$$

$$F_{m_2 \text{ on } m_3} = \frac{gm_2m_3}{r_{23}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10.0\ \text{kg})(5.0\ \text{kg})}{(0.20\ \text{m})^2 + (0.10\ \text{m})^2} = 6.67\times10^{-8}\ \text{N}$$

$$\begin{aligned}\vec{F}_{m_2 \text{ on } m_3} &= (6.67\times10^{-8}\ \text{N})\sin\phi\hat{i} - (6.67\times10^{-8}\ \text{N})\cos\phi\hat{j}\\ &= (6.67\times10^{-8}\ \text{N})\left(\frac{10\ \text{cm}}{22.36\ \text{cm}}\right)\hat{i} - (6.67\times10^{-8}\ \text{N})\left(\frac{20\ \text{cm}}{22.36\ \text{cm}}\right)\hat{j}\\ &= (2.983\times10^{-8}\ \text{N})\hat{i} - (5.966\times10^{-8}\ \text{N})\hat{j}\end{aligned}$$

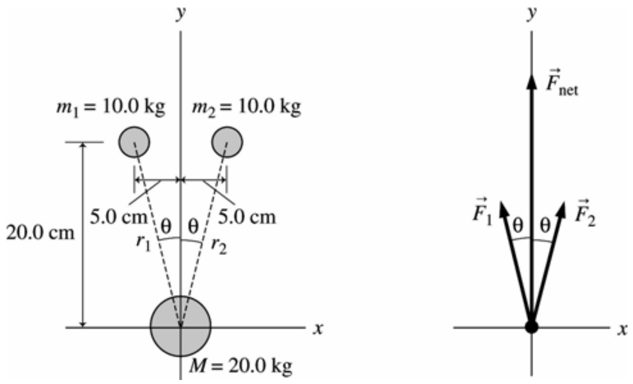
$$\vec{F}_{\text{on } m_3} = \vec{F}_{m_1 \text{ on } m_3} + \vec{F}_{m_2 \text{ on } m_3} = 2.983\times10^{-8}\hat{i}\ \text{N} - 2.264\times10^{-7}\hat{j}\ \text{N}$$

$$F_{\text{on } m_3} = \sqrt{(2.983\times10^{-8}\ \text{N})^2 + (-2.264\times10^{-7}\ \text{N})^2} = 2.284\times10^{-7}\ \text{N}$$

$$\theta' = \tan^{-1}\left(\frac{(\vec{F}_{\text{on } m_3})_x}{(\vec{F}_{\text{on } m_3})_y}\right) = \tan^{-1}\left(\frac{2.983\times10^{-8}\ \text{N}}{2.264\times10^{-7}\ \text{N}}\right) = 7.51°$$

Thus $\vec{F}_{\text{on } m_3} = 2.3\times10^{-7}$ N, 7.5° ccw from the $-y$-axis.

13.36. Visualize:



Solve: The angle $\theta = \tan^{-1}\left(\frac{5}{20}\right) = 14.04°$. The distance $r_1 = r_2 = \sqrt{(0.050\text{ m})^2 + (0.200\text{ m})^2} = 0.206\text{ m}$. The forces on the 20.0 kg mass are

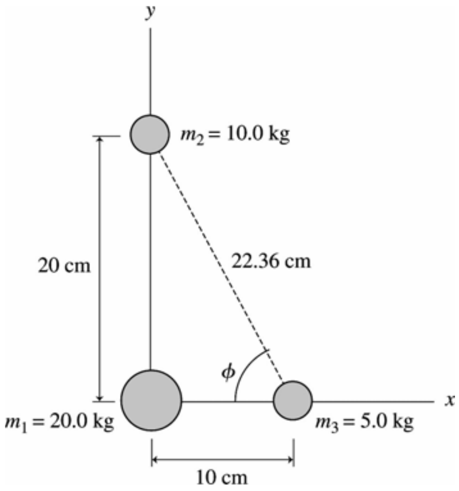
$$\vec{F}_1 = G\frac{Mm_1}{r_1^2}(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$\vec{F}_2 = G\frac{Mm_2}{r_2^2}(\sin\theta\hat{i} + \cos\theta\hat{j})$$

Note $m_1 = m_2$ and $r_1 = r_2$. Thus, the net force on the 20.0 kg mass is

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 = 2G\frac{Mm_1}{r_1^2}\cos\theta\hat{j} \\ &= \frac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(20.0\text{ kg})(5.0\text{ kg})\cos(14.04°)}{(0.206\text{ m})^2}\hat{j} \\ &= 3.0\times10^{-7}\ \hat{j}\text{ N}\end{aligned}$$

13.37. Visualize:



Solve: The total gravitational potential energy is the sum of the potential energies due to the interactions of the pairs of masses.

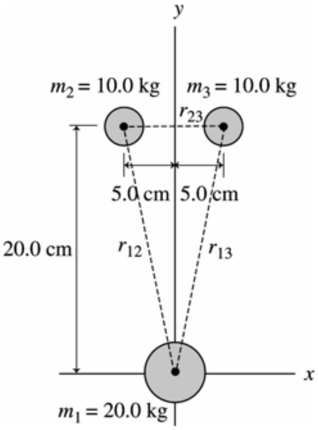
$$U = U_{12} + U_{13} + U_{23}$$
$$= -G\frac{m_1 m_2}{r_{12}} - G\frac{m_1 m_3}{r_{13}} - G\frac{m_2 m_3}{r_{23}}$$

With $m_1 = 20.0 \text{ kg}, m_2 = 10.0 \text{ kg}, m_3 = 5.0 \text{ kg}, r_{12} = 0.20 \text{ m}, r_{13} = 0.10 \text{ m},$ and $r_{23} = 0.2236 \text{ m},$

$$U = -1.48 \times 10^{-7} \text{ J}$$

Assess: The gravitational potential energy is negative because the masses attract each other. It is a scalar, so there are no vector calculations required.

13.38. Visualize:



Solve: The total gravitational potential energy is the sum of the potential energies due to the interactions of the pairs of masses.

$$U = U_{12} + U_{13} + U_{23}$$
$$= -G\frac{m_1 m_2}{r_{12}} - G\frac{m_1 m_3}{r_{13}} - G\frac{m_2 m_3}{r_{23}}$$

With $m_1 = 20.0 \text{ kg}, m_2 = m_3 = 10.0 \text{ kg}, r_{12} = r_{13} = \sqrt{(0.050 \text{ m})^2 + (0.200 \text{ m})^2} = 0.206 \text{ m},$ and $r_{23} = 0.100 \text{ m},$

$$U = -1.96 \times 10^{-7} \text{ J}$$

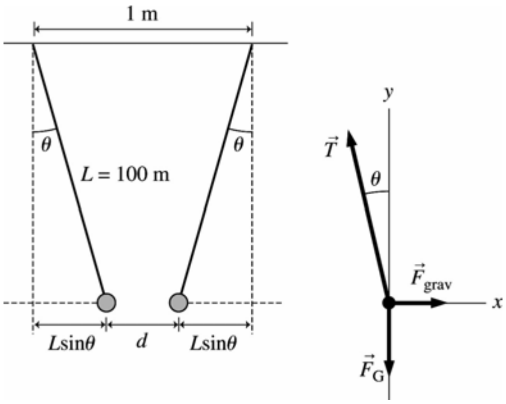
Assess: The gravitational potential energy is negative because the masses attract each other. It is a scalar, so there are no vector calculations required.

13.39. Solve: We are given $M_1 + M_2 = 150 \text{ kg}$, which means $M_1 = 150 \text{ kg} - M_2$. We also have

$$\frac{GM_1M_2}{(0.20 \text{ m})^2} = 8.00 \times 10^{-6} \text{ N} \quad \Rightarrow \quad M_1M_2 = \frac{(8.00 \times 10^{-6} \text{ N})(0.20 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 4798 \text{ kg}^2$$

Thus, $(150 \text{ kg} - M_2)M_2 = 4798 \text{ kg}^2$ or $M_2^2 - (150 \text{ kg})M_2 + (4798 \text{ kg}^2) = 0.$ Solving this equation gives $M_2 = 103.75 \text{ kg}$ and $46.25 \text{ kg}.$ The two masses are 104 and 46 kg.

13.40. Visualize: Because of the gravitational force of attraction between the lead spheres, the cables will make an angle of θ with the vertical. The distance between the sphere centers is therefore going to be less than 1 m. The free-body diagram shows the forces acting on the lead sphere.



Solve: We can see from the diagram that the distance between the centers is $d = 1.000\text{ m} - 2L\sin\theta$. Each sphere is in static equilibrium, so Newton's second law is

$$\sum F_x = F_{\text{grav}} - T\sin\theta = 0 \quad\Rightarrow\quad T\sin\theta = F_{\text{grav}}$$
$$\sum F_y = T\cos\theta - mg = 0 \quad\Rightarrow\quad T\cos\theta = mg$$

Dividing these two equations to eliminate the tension *T* yields

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{F_{\text{grav}}}{mg} = \frac{Gmm/d^2}{mg} = \frac{Gm}{d^2 g}$$

We know that *d* is going to be only very, very slightly less than 1.00 m. The very slight difference is not going to be enough to affect the value of F_{grav}, the gravitational attraction between the two masses, so we'll evaluate the right side of this equation by using 1.00 m for *d*. This gives

$$\tan\theta = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}{(1.00\text{ m})^2(9.81\text{ m/s}^2)} = 6.80\times10^{-10} \quad\Rightarrow\quad \theta = (3.90\times10^{-8})^\circ$$

This small angle causes the two spheres to move closer by $2L\sin\theta = 1.4\times10^{-7}$ m.

13.41. Model: Use Newton's law of gravitation.
Visualize: Objects will begin to fall up when the gravitational force from the black hole has the same magnitude as the gravitational force from the earth. Let *d* be the distance from the object (subscript o) on the earth's surface to the black hole.
Solve:

$$G\frac{m_{\text{o}}M_{\text{e}}}{R_{\text{e}}^2} = G\frac{m_{\text{o}}(12M_{\text{S}})}{d^2} \Rightarrow \frac{d^2}{R_{\text{e}}^2} = \frac{Gm_{\text{o}}(12M_{\text{S}})}{Gm_{\text{o}}M_{\text{e}}} = \frac{12M_{\text{S}}}{M_{\text{e}}} \Rightarrow$$
$$\frac{d}{R_{\text{e}}} = \sqrt{\frac{12M_{\text{S}}}{M_{\text{e}}}} = \sqrt{\frac{12(1.99\times10^{30}\text{ kg})}{5.98\times10^{24}\text{ kg}}} = 1998$$

We must add one more earth radius to the result to get the center-to-center distance (since the object was on the earth's surface), but that doesn't change the answer to three significant figures. The distance from the black hole to the center of the earth is $2000R_{\text{e}}$ to three significant figures.

Assess: $2000R_{\text{e}}$ is less than a tenth of one astronomical unit.

13.42. Model: Consider the object as a particle and take the planet to be a spherical mass.
Solve: Conservation of mechanical energy of the object gives

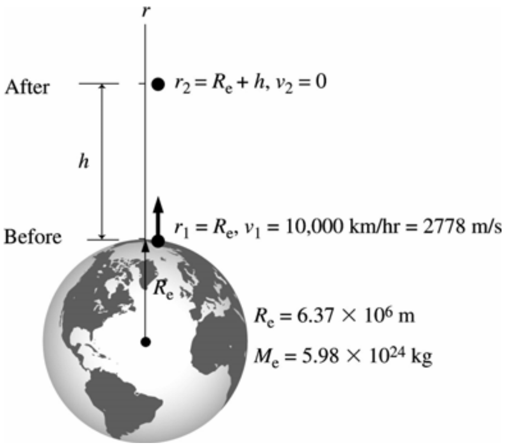
$$-G\frac{Mm}{R+h} = \frac{1}{2}mv^2 - G\frac{Mm}{R}$$

The object's mass drops out. Solving for the speed as it hits the ground,

$$v=\sqrt{2GM\left(\frac{1}{R}-\frac{1}{R+h}\right)}=\sqrt{2GM\left(\frac{R+h-R}{R(R+h)}\right)}=\sqrt{\frac{2GMh}{R(R+h)}}$$

Assess: Compare this to $v=\sqrt{2gh}=\sqrt{\dfrac{2GMh}{R^2}}$, which is the result if the potential $U=mgh$ is used.

13.43. **Model:** Model the earth and the projectile as spherical masses. Ignore air resistance. This is an isolated system, so mechanical energy is conserved.
Visualize:



r
After
$r_2 = R_e + h,\ v_2 = 0$
h
Before
$r_1 = R_e,\ v_1 = 10{,}000$ km/hr = 2778 m/s
R_e
$R_e = 6.37 \times 10^6$ m
$M_e = 5.98 \times 10^{24}$ kg

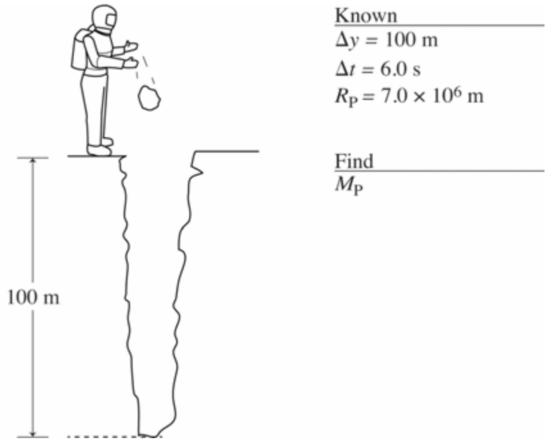
A pictorial representation of the before-and-after events is shown.
Solve: After using $v_2 = 0$ m/s, the energy conservation equation $K_2+U_2=K_1+U_1$ is

$$0\text{ J}-\frac{GM_e m_p}{R_e+h}=\frac{1}{2}m_p v_1^2-\frac{GM_e m_p}{R_e}$$

The projectile mass cancels. Solving for *h*, we find

$$h=\left[\frac{1}{R_e}-\frac{v_1^2}{2GM_e}\right]^{-1}-R_e=4.2\times10^5\text{m}$$

13.44. **Model:** Model the rock as a particle and assume constant acceleration so we can use the kinematic equations.
Visualize:



Solve: First apply the kinematic equations.

$$\Delta y = \tfrac{1}{2} g(\Delta t)2 \Rightarrow g = \frac{2\Delta y}{(\Delta t)^2}$$

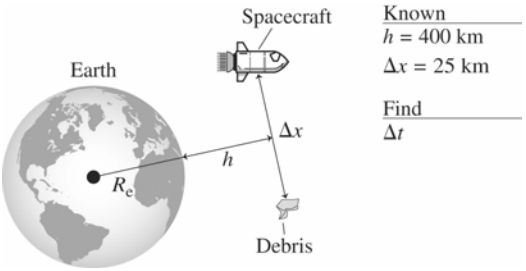
We also apply Newton's law of gravitation: $g = GM_\text{P}/R_\text{P}^2$. Set the two equations for g equal to each other.

$$\frac{2\Delta y}{(\Delta t)^2} = G\frac{M_\text{P}}{R_\text{P}^2} \Rightarrow M_\text{P} = \frac{2\Delta y}{(\Delta t)^2}\frac{R_\text{P}^2}{G} = \frac{2(100\text{ m})}{(6.0\text{ s})^2}\frac{(7.0\times10^6\text{ m})^2}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 4.1\times10^{24}\text{ kg}$$

Assess: This planet has almost as much mass as Venus. By the way, there is a terrestrial-sized exoplanet orbiting the star Alpha Centauri B, but we have not yet dropped rocks in crevasses on it.

13.45. Model: Since the distance between the spacecraft and the debris is very much smaller than the circumference of the orbit we will approximate the motion as straight line motion along the *x* axis.

Visualize: In the astronaut's reference frame the collision will be at a speed of $2v_\text{orbit}$.



Solve: First apply the kinematic equations.

$$v_\text{orbit} = \sqrt{\frac{GM_\text{e}}{R_\text{e}+h}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{6.37\times10^6\text{ m}+400{,}000\text{ m}}} = 7.68\text{ km/s}$$

Now determine the time before collision.

$$\Delta t = \frac{\Delta x}{v} = \frac{\Delta x}{2v_\text{orbit}} = \frac{25\text{ km}}{2(7.68\text{ km/s})} = 1.6\text{ s}$$

Assess: These numbers are fairly realistic; there won't be much time for a course correction unless the debris is discovered and tracked a lot earlier (farther away).

13.46. Model: Model the asteroid as a spherical mass and yourself as a point mass. This is an isolated system, so mechanical energy is conserved.

Visualize: The radius of the asteroid is R_a and its mass is M_a.

Solve: We first calculate your jumping speed on the earth. The conservation of energy equation gives

$$U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f} \Rightarrow 0\text{ J} + \tfrac{1}{2}mv_\text{i}^2 = mg\Delta y + 0\text{ J}$$

$$0\text{ J} + \frac{1}{2}mv_\text{i}^2 = mg(0.45\text{ m}) \quad\Rightarrow\quad v_\text{i} = \sqrt{2(9.8\text{ m/s}^2)(0.45\text{ m})} = 2.97\text{ m/s}$$

The conservation of energy equation $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$ for the asteroid gives

$$-\frac{GM_\text{a}m}{R_\text{a}} + \frac{1}{2}mv_\text{i}^2 = -\frac{GM_\text{a}m}{R_\text{a}+r} + \frac{1}{2}mv_\text{f}^2$$

The minimum speed for escape is the one that will cause you to stop only when the separation between you and the asteroid becomes very large. Noting that $v_\text{f} \to 0$ m/s as $r \to \infty$, the whole right side is zero. Use $M_\text{a} = \rho\tfrac{4}{3}\pi R_\text{a}^3$ and rearrange to get

$$v_\text{i}^2 = \frac{2GM_\text{a}}{R_\text{a}} = \frac{2G\rho\frac{4}{3}\pi R_\text{a}^3}{R_\text{a}} = \tfrac{8}{3}G\rho\pi R_\text{a}^2$$

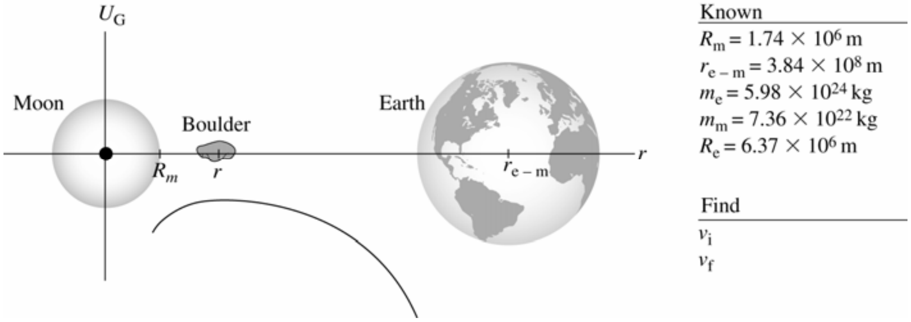
Solve for R_a.

$$R_\text{a} = \sqrt{\frac{v_\text{i}^2}{\frac{8}{3}G\rho\pi}} = \sqrt{\frac{(2.97\text{ m/s})^2}{\frac{8}{3}(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2800\text{ kg/m}^3)\pi}} = 2.37\text{ km}$$

The maximum diameter of asteroid you could escape by jumping is twice this radius, or 4.7 km.

Assess: This is a typical size for an asteroid.

13.47. Model: Model the boulders as particles. Ignore air resistance and rotational and orbital motion. The earth-moon-boulder system is an isolated system, so mechanical energy is conserved.
Visualize: Set up a coordinate system with the origin at the center of the moon. Let r be the location of the boulder when its kinetic energy is zero.


U_G
Moon
Boulder
Earth
R_m
r
r_{e-m}
r
Known
$R_m = 1.74 \times 10^6$ m
$r_{e-m} = 3.84 \times 10^8$ m
$m_e = 5.98 \times 10^{24}$ kg
$m_m = 7.36 \times 10^{22}$ kg
$R_e = 6.37 \times 10^6$ m
Find
v_i
v_f

Solve: (a) We first calculate r, the height the boulder must be launched to get out of the moon's gravity well and into the earth's. This will be where the potential energy is a maximum (and the kinetic energy is zero).

$$U_\text{G} = -G\left(\frac{m_\text{e}m_\text{b}}{r_\text{e-b}} + \frac{m_\text{m}m_\text{b}}{r_\text{m-b}} + \frac{m_\text{e}m_\text{m}}{r_\text{e-m}}\right)$$

To find where this is a maximum, take the derivative with respect to r and set to zero. Let $r_\text{m-b} = r$ and $r_\text{e-b} = r_\text{e-m} - r$.

$$\frac{dU_\text{G}}{dr} = G\left(\frac{-m_\text{e}m_\text{b}}{(r_\text{e-m}-r)^2} + \frac{m_\text{m}m_\text{b}}{r^2}\right) = 0$$

Cancel G and m_b.

$$\frac{m_\text{e}}{(r_\text{e-m}-r)^2} = \frac{m_\text{m}}{r^2} \Rightarrow \left(1-\frac{m_\text{e}}{m_\text{m}}\right)r^2 - 2r_\text{e-m}r + r^2 = 0$$

Use the quadratic formula to solve for r.

$$r = r_\text{e-m}\frac{1\pm\sqrt{m_\text{e}/m_\text{m}}}{1-m_\text{e}/m_\text{m}} = (3.84\times10^8\text{ m})\frac{1\pm\sqrt{(5.98\times10^{24}\text{ kg})/(7.36\times10^{22}\text{ kg})}}{1-(5.98\times10^{24}\text{ kg})/(7.36\times10^{22}\text{ kg})}$$

There are two solutions to this equation, but we want the one between the moon and the earth—that is, the positive value of r in our coordinate system.

$$r = 0.09986r_\text{e-m} = 3.835\times10^7\text{ m}$$

Now use conservation of mechanical energy for the three-body system: $U_\text{i} + K_\text{i} = U_\text{f} + K_\text{f}$ where K_f is zero. The initial location of the boulder is on the moon's surface at R_m; the final location of the boulder is at r.

$$K_\text{i} = -U_\text{i} + U_\text{f}$$

$$\tfrac{1}{2}m_\text{b}v_\text{i}^2 = G\left(\frac{m_\text{e}m_\text{m}}{r_\text{e-m}} + \frac{m_\text{e}m_\text{b}}{(r_\text{e-m}-R_\text{m})} + \frac{m_\text{m}m_\text{b}}{R_\text{m}} - \frac{m_\text{e}m_\text{m}}{r_\text{e-m}} - \frac{m_\text{e}m_\text{b}}{(r_\text{e-m}-r)} - \frac{m_\text{m}m_\text{b}}{r}\right)$$

The earth-moon terms don't change after the boulder is launched, so they cancel each other. We then cancel m_b from both sides of the equation.

$$\frac{1}{2}v_i^2 = G\left(\frac{m_e}{(r_{e\text{-}m} - R_m)} + \frac{m_m}{R_m} - \frac{m_e}{(r_{e\text{-}m} - r)} - \frac{m_m}{r}\right)$$

Rearrange terms, multiply by 2, and take square roots.

$$v_i = \sqrt{2G\left(m_e\left(\frac{1}{(r_{e\text{-}m} - R_m)} - \frac{1}{(r_{e\text{-}m} - r)}\right) + m_m\left(\frac{1}{R_m} - \frac{1}{r}\right)\right)}$$

Plug in the numbers.

$$v_i = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(-1.6569\times10^{15}\ \text{kg/m} + 4.038\times10^{16}\ \text{kg/m})} = 2.273\ \text{km/s}$$

The boulders must be launched at 2.3 km/s to make it to the earth. This is less than the escape speed.

(b) To find the impact speed on the earth's surface we employ the same energy conservation technique again, this time with the initial speed zero. In this instance the initial position is r and the final position is on the earth's surface at $r_{e\text{-}m} - R_e$.

$$K_f = -U_f + U_i$$

$$\frac{1}{2}m_b v_f^2 = G\left(\frac{m_e m_m}{r_{e\text{-}m}} + \frac{m_e m_b}{R_e} + \frac{m_m m_b}{(r_{e\text{-}m} - R_e)} - \frac{m_e m_m}{r_{e\text{-}m}} - \frac{m_e m_b}{(r_{e\text{-}m} - r)} - \frac{m_m m_b}{r}\right)$$

The earth-moon terms don't change as the boulder falls so they cancel each other. We then cancel m_b from both sides of the equation.

$$\frac{1}{2}v_f^2 = G\left(\frac{m_e}{R_e} + \frac{m_m}{(r_{e\text{-}m} - R_e)} - \frac{m_e}{(r_{e\text{-}m} - r)} - \frac{m_m}{r}\right)$$

Rearrange terms, multiply by 2, and take square roots.

$$v_f = \sqrt{2G\left(m_e\left(\frac{1}{R_e} - \frac{1}{(r_{e\text{-}m} - r)}\right) + m_m\left(\frac{1}{(r_{e\text{-}m} - R_e)} - \frac{1}{r}\right)\right)}$$

Plug in the numbers.

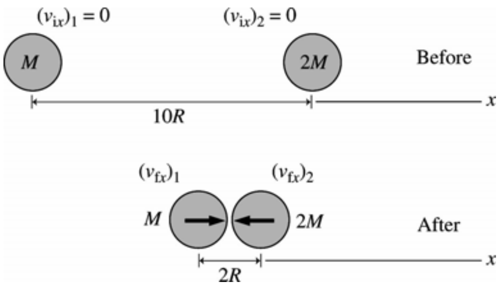
$$v_i = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.2147\times10^{17}\ \text{kg/m} - 1.724\times10^{15}\ \text{kg/m})} = 11\ \text{km/s}$$

The boulders' impact speed is 11 km/s ignoring air resistance.

Assess: The value of r that we found is also the point where the gravitational force of the moon on the boulder has the same magnitude as the gravitational force of the earth on the boulder. This makes sense because $F_G = -dU_G/dr$. The required launch speed is less than the moon's escape speed, as expected, because the boulder only needs to get over the potential energy hill, not all the way out to infinity. The impact speed is almost the earth's escape speed, however.

13.48. Model: The two asteroids make an isolated system, so mechanical energy is conserved. We will also use the law of conservation of momentum for our system.

Visualize:



Solve: The conservation of momentum equation $p_{\text{fx}} = p_{\text{ix}}$ is

$$M(v_{\text{fx}})_1 + 2M(v_{\text{fx}})_2 = 0 \text{ kg m/s} \quad \Rightarrow \quad (v_{\text{fx}})_1 = -2(v_{\text{fx}})_2$$

The equation for mechanical energy conservation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ is

$$\frac{1}{2}M(v_{\text{fx}})_1^2 + \frac{1}{2}(2M)(v_{\text{fx}})_2^2 - \frac{G(M)(2M)}{2R} = -\frac{G(M)(2M)}{10R} \quad \Rightarrow \quad \frac{1}{2}[2(v_{\text{fx}})_2]^2 + (v_{\text{fx}})_2^2 = \frac{0.8GM}{R}$$

$$(v_{\text{fx}})_2 = -0.516\sqrt{\frac{GM}{R}} \quad \Rightarrow \quad (v_{\text{fx}})_1 = -2(v_{\text{fx}})_2 = 1.032\sqrt{\frac{GM}{R}}$$

The heavier asteroid has a speed of $0.516(GM/R)^{1/2}$ and the lighter one a speed of $1.032(GM/R)^{1/2}$.

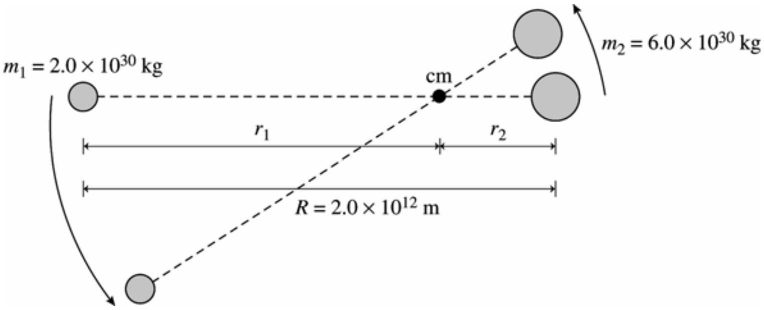
13.49. Model: Model the distant planet (P) as a spherical mass.
Solve: The acceleration at the surface of the planet and at the altitude *h* are

$$g_{\text{surface}} = \frac{GM_\text{P}}{R^2} \quad \text{and} \quad g_{\text{altitude}} = \frac{1}{2}g_{\text{surface}} \Rightarrow G\frac{M_\text{P}}{(R+h)^2} = \frac{1}{2}\frac{GM_\text{P}}{R^2}$$

$$(R+h)^2 = 2R^2 \quad \Rightarrow \quad R+h = \sqrt{2}R \quad \Rightarrow \quad h = (\sqrt{2}-1)R = 0.414R$$

That is, the starship is orbiting at an altitude of 0.414*R*.

13.50. Model: The stars are spherical masses. They each rotate about the system's center of mass.
Visualize:



$m_1 = 2.0 \times 10^{30}$ kg
cm
$m_2 = 6.0 \times 10^{30}$ kg
r_1
r_2
$R = 2.0 \times 10^{12}$ m

Solve: (a) The stars rotate about the system's center of mass with the same period: $T_1 = T_2 = T$. We can locate the center of mass by letting the origin be at the smaller-mass star. Then

$$r_1 = r_{\text{cm}} = \frac{(2.0\times10^{30}\text{ kg})(0\text{ m}) + (6.0\times10^{30}\text{ kg})(2.0\times10^{12}\text{ m})}{2.0\times10^{30}\text{ kg} + 6.0\times10^{30}\text{ kg}} = 1.5\times10^{12}\text{ m}$$

Mass m_2 undergoes uniform circular motion with radius $r_2 = 0.5\times10^{12}$ m due to the gravitational force of mass m_1 at distance $R = 2.0\times10^{12}$ m. The gravitational force is responsible for the centripetal acceleration, so

$$F_{\text{grav}} = \frac{Gm_1m_2}{R^2} = m_2a_{\text{centripetal}} = \frac{m_2v_2^2}{r_2} = \frac{m_2}{r_2}\left(\frac{2\pi r_2}{T}\right)^2 = \frac{4\pi^2m_2r_2}{T^2}$$

$$T = \left[\frac{4\pi^2}{Gm_1}r_2R^2\right]^{1/2} = \left[\frac{4\pi^2(0.5\times10^{12}\text{ m})(0.5\times10^{12}\text{ m})^2}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{ kg})}\right]^{1/2} = 7.693\times10^8\text{ s} = 24\text{ years}$$

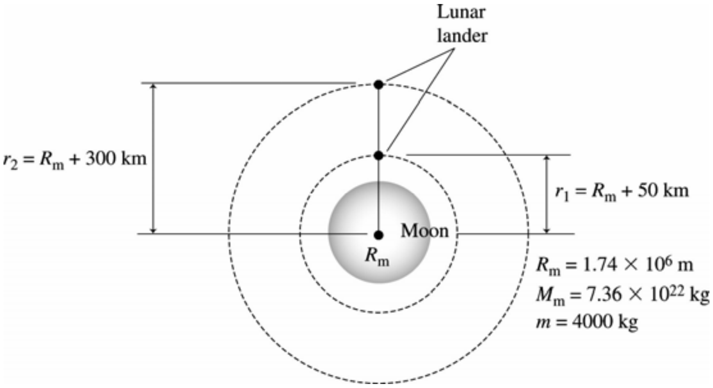
(b) The speed of each star is $v = (2\pi r)/T$. Thus

$$v_1 = \frac{2\pi r_1}{T} = \frac{2\pi(1.5\times10^{12}\text{ m})}{7.693\times10^8\text{ s}} = 12.3\text{ km/s}$$

$$v_2 = \frac{2\pi r_2}{T} = \frac{2\pi(0.5\times10^{12}\text{ m})}{7.693\times10^8\text{ s}} = 4.1\text{ km/s}$$

13.51. Model: Model the moon (m) as a spherical mass and the lander (l) as a particle. This is an isolated system, so mechanical energy is conserved.

Visualize: The initial position of the lunar lander $(\text{mass} = m_1)$ is at a distance $r_1 = R_\text{m} + 50\text{ km}$ from the center of the moon. The final position of the lunar lander is the orbit whose distance from the center of the moon is $r_2 = R_\text{m} + 300\text{ km}$.


Lunar lander
$r_2 = R_\text{m} + 300$ km
$r_1 = R_\text{m} + 50$ km
Moon
R_m
$R_\text{m} = 1.74 \times 10^6$ m
$M_\text{m} = 7.36 \times 10^{22}$ kg
$m = 4000$ kg

Solve: The external work done by the thrusters is

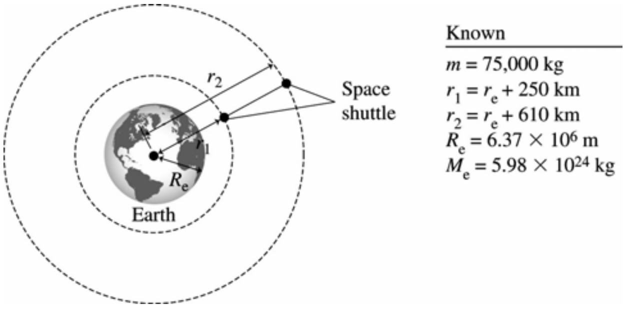
$$W_\text{ext} = \Delta E_\text{mech} = \tfrac{1}{2}\Delta U_\text{g}$$

where we used $E_\text{mech} = \tfrac{1}{2}U_\text{g}$ for a circular orbit. The change in potential energy is from the initial orbit at $r_\text{i} = R_\text{m} + 50\text{ km}$ to the final orbit $r_\text{f} = R_\text{m} + 300\text{ km}$. Thus

$$\begin{aligned} W_\text{ext} &= \frac{1}{2}\left(\frac{-GM_\text{m}m}{r_\text{f}} - \frac{-GM_\text{m}m}{r_\text{i}}\right) = \frac{GM_\text{m}m}{2}\left(\frac{1}{r_\text{i}} - \frac{1}{r_\text{f}}\right) \\ &= \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\text{ kg})(4000\text{ kg})}{2}\left(\frac{1}{1.79\times10^6\text{ m}} - \frac{1}{2.04\times10^6\text{ m}}\right) \\ &= 6.7\times10^8\text{ J} \end{aligned}$$

13.52. Model: Model the earth (e) as a spherical mass and the space shuttle (s) as a point particle. This is an isolated system, so the mechanical energy is conserved.

Visualize: The space shuttle $(\text{mass} = m_\text{s})$ is at a distance of $r_1 = R_\text{e} + 250\text{ km}$.


r_2
Space shuttle
r_1
R_e
Earth
Known
$m = 75{,}000$ kg
$r_1 = r_\text{e} + 250$ km
$r_2 = r_\text{e} + 610$ km
$R_\text{e} = 6.37 \times 10^6$ m
$M_\text{e} = 5.98 \times 10^{24}$ kg

Solve: The external work done by the thrusters is

$$W_\text{ext} = \Delta E_\text{mech} = \tfrac{1}{2}\Delta U_\text{g}$$

where we used $E_\text{mech} = \tfrac{1}{2}U_\text{g}$ for a circular orbit. The change in potential energy is from the initial orbit at $r_1 = R_\text{e} + 250\text{ km}$ to the final orbit $r_\text{f} = R_\text{e} + 610\text{ km}$. Thus

$$W_{\text{ext}} = \frac{1}{2}\left(\frac{-GM_{\text{e}}m}{r_{\text{f}}} - \frac{-GM_{\text{e}}m}{r_{\text{i}}}\right) = \frac{GM_{\text{e}}m}{2}\left(\frac{1}{r_{\text{i}}} - \frac{1}{r_{\text{f}}}\right)$$
$$= \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(75{,}000\ \text{kg})}{2}\left(\frac{1}{6.62\times10^6\ \text{m}} - \frac{1}{6.98\times10^6\ \text{m}}\right)$$
$$= 1.2\times10^{11}\ \text{J}$$

This much energy must be supplied by burning the on-board fuel.

13.53. Model: Assume the earth's current orbit is circular.
Visualize: Follow Example 13.6 in the textbook.
Solve:

$$\Delta E_{\text{mech}} = \tfrac{1}{2}\Delta U_{\text{G}} = \frac{1}{2}(-Gm_{\text{s}}m_{\text{e}})\left(\frac{1}{r_{\text{s-e}}+d} - \frac{1}{r_{\text{s-e}}}\right)$$
$$= \frac{(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{2}$$
$$\left(\frac{1}{1.50\times10^{11}\ \text{m}+1000\ \text{m}} - \frac{1}{1.50\times10^{11}\ \text{m}}\right)$$
$$= 1.8\times10^{25}\ \text{J}$$

If your calculator doesn't have enough significant digits then use the binomial approximation:

$$\left(\frac{1}{r_{\text{s-e}}+d} - \frac{1}{r_{\text{s-e}}}\right) = \frac{1}{r_{\text{s-e}}}\left(\left(1+\frac{d}{r_{\text{s-e}}}\right)^{-1} - 1\right) \approx \frac{1}{r_{\text{s-e}}}\left(1-\frac{d}{r_{\text{s-e}}}-1\right) = \frac{-d}{r_{\text{s-e}}^2}$$

Plug this approximation into the energy equation.

$$\Delta E_{\text{mech}} = \tfrac{1}{2}\Delta U_{\text{G}} \approx \frac{1}{2}(-Gm_{\text{s}}m_{\text{e}})\left(\frac{-d}{r_{\text{s-e}}^2}\right)$$
$$= \frac{(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{2}\frac{(-1000\ \text{m})}{(1.50\times10^{11}\text{m})^2}$$
$$= 1.8\times10^{25}\ \text{J}$$

Assess: This is a lot of joules!

13.54. Model: Model the moon as a spherical mass and the satellite as a point mass.
Visualize: The rotational period of the satellite is the same as the rotational period of the moon around its own axis. This time happens to be 27.3 days.
Solve: The gravitational force between the moon and the satellite provides the centripetal acceleration necessary for circular motion around the moon. Therefore,

$$\frac{GM_{\text{m}}m}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$
$$r^3 = \frac{GM_{\text{m}}T^2}{4\pi^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\ \text{kg})(27.3\times24\times3600\ \text{s})^2}{4\pi^2}$$
$$r = 8.84\times10^7\ \text{m}$$

Since $r = R_{\text{m}} + h,$ then $h = r - R_{\text{m}} = 8.84\times10^7\ \text{m} - 1.74\times10^6\ \text{m} = 8.67\times10^7\ \text{m}.$

13.55. Model: Assume the earth's current orbit is circular.
Visualize: Let *M* be the mass of the comet and *m* the mass of the lander and *R* the radius of the comet.
Solve: (a)

$$v = \frac{2\pi r}{T} = \frac{2\pi(25\ \text{km})}{11\ \text{d}}\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 0.1653\ \text{m/s}$$

We report this as 0.17 m/s.

(b)

$$v=\sqrt{\frac{GM}{r_{\text{orbit}}}}\Rightarrow M=\frac{r_{\text{orbit}}}{G}v^2=\frac{25\text{ km}}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}(0.1653\text{ m/s})^2=1.0\times10^{13}\text{ kg}$$

(c) Use conservation of energy with the initial kinetic energy equal to zero.

$$K_{\text{f}}=-U_{\text{f}}+U_{\text{i}}$$

$$\tfrac{1}{2}mv_{\text{f}}^2=G\left(\frac{Mm}{R}-\frac{Mm}{r_{\text{orbit}}}\right)$$

Cancel *m*, multiply by 2 and take square roots.

$$v_{\text{f}}=\sqrt{2GM\left(\frac{1}{R}-\frac{1}{r_{\text{orbit}}}\right)}$$

$$=\sqrt{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{13}\text{ kg})\left(\frac{1}{1.8\text{ km}}-\frac{1}{25\text{ km}}\right)}=0.84\text{ m/s}$$

Assess: This is a slow speed because the comet's mass is so small.

13.56. Model: Model the earth as a spherical mass and the satellite as a point mass.

Visualize: The satellite is directly over a point on the equator once every two days. Thus, $T=2T_{\text{e}}=2\times24\times 3600\text{ s}=1.728\times10^5\text{ s}$.

Solve: From Kepler's laws applied to a circular orbit:

$$T^2=\left(\frac{4\pi^2}{GM_{\text{e}}}\right)r^3$$

$$r^3=\frac{GM_{\text{e}}T^2}{4\pi^2}=\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(1.728\times10^5)^2}{4\pi^2}$$

$$r=6.71\times10^7\text{ m}$$

Assess: The radius of the orbit is larger than the geosynchronous orbit.

13.57. Visualize: Please refer to Figure P13.57.

Solve: The gravitational force on one of the masses is due to the star *and* the other planet. Thus

$$G\frac{Mm}{r^2}+\frac{Gmm}{(2r)^2}=\frac{mv^2}{r}=\frac{m}{r}\left(\frac{2\pi r}{T}\right)^2\quad\Rightarrow\quad\frac{GM}{r}+\frac{Gm}{4r}=\frac{4\pi^2r^2}{T^2}$$

$$\frac{G}{r}\left(M+\frac{m}{4}\right)=\frac{4\pi^2r^2}{T^2}\quad\Rightarrow\quad T=\left[\frac{4\pi^2r^3}{G}\frac{1}{(M+m/4)}\right]^{1/2}$$

13.58. Solve: (a) Taking the logarithm of both sides of $v^p=Cu^q$ gives

$$[\log(v^p)=p\log v]=[\log(Cu^q)=\log C+q\log u]\quad\Rightarrow\quad\log v=\frac{q}{p}\log u+\frac{\log C}{p}$$

But $x=\log u$ and $y=\log v$, so *x* and *y* are related by

$$y=\left(\frac{q}{p}\right)x+\frac{\log C}{p}$$

(b) The previous result shows there is a linear relationship between *x* and *y*, so there is a linear relationship between log *u* and log *v*. The graph of a linear relationship is a straight line, so the graph of log *v*-versus-log *u* will be a straight line.

(c) The slope of the straight line represented by the equation $y=(q/p)x+\log C/p$ is q/p. Thus, the slope of the log v-versus-log u graph will be q/p.

(d) The predicted y-intercept of the graph is (log C)/p, and the experimentally determined value is 9.264. Equating these, we can solve for M. Because the planets all orbit the sun, the mass we are finding is $M=M_{\text{sun}}$.

$$\frac{1}{2}\log C=\frac{1}{2}\log\left(\frac{4\pi^2}{GM_{\text{sun}}}\right)=-9.264 \quad\Rightarrow\quad \frac{4\pi^2}{GM_{\text{sun}}}=10^{-18.528}=\frac{1}{10^{18.528}}$$

$$M_{\text{sun}}=\frac{4\pi^2}{G}\cdot 10^{18.528}=1.996\times10^{30}\text{ kg}$$

Assess: The tabulated value, to three significant figures, is $M_{\text{sum}}=1.99\times10^{30}$ kg. We have used the orbits of the planets to "weigh the sun!"

13.59. Solve: (a) Dividing the circumference of the orbit by the period gives

$$v=\frac{2\pi R_{\text{s}}}{T}=\frac{2\pi(1.0\times10^4\text{ m})}{1.0\text{ s}}=6.3\times10^4\text{ m/s}$$

(b) Using the formula for the acceleration at the surface,

$$g_{\text{surface}}=\frac{GM_{\text{s}}}{R_{\text{s}}^2}=\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{(1.0\times10^4\text{ m})^2}=1.3\times10^{12}\text{ m/s}^2$$

(c) The mass of an object on the earth will be the same as its mass on the star. The gravitational force is

$$(F_{\text{G}})_{\text{star}}=mg_{\text{ surface}}=1.3\times10^{12}\text{N}$$

(d) The radius of the orbit of the satellite is $r=1\times10^4\text{ m}+1.0\times10^3\text{ m}=1.1\times10^4\text{ m}$. The period is

$$T^2=\frac{4\pi^2r^3}{GM_{\text{s}}}=\frac{4\pi^2(1.1\times10^4\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}\quad\Rightarrow\quad T=6.29\times10^{-4}\text{ s}$$

This means there are 1589 revolutions per second or 9.5×10^4 orbits per minute.

(e) Applying Equation 13.25 for a geosynchronous orbit,

$$r^3=\frac{GM_{\text{s}}}{4\pi^2}T^2=\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(1.0\text{ s})}{4\pi^2}\quad\Rightarrow\quad r=1.5\times10^6\text{ m}$$

13.60. Model: Assume the solar system is a point particle.

Solve: (a) The radius of the orbit of the solar system in the galaxy is 25,000 light years. This means

$$r=25{,}000\text{ light years}=2500(3.0\times10^8\text{ m / s})(365\text{ d/y})(24\text{ h/d})(3600\text{ s/h})(1\text{ y})=2.36\times10^{20}\text{ m}$$

$$T=\frac{2\pi r}{v}=\frac{2\pi(2.36\times10^{20}\text{ m})}{2.30\times10^5\text{ m/s}}=6.64\times10^{15}\text{ s}=2.1\times10^8\text{ years}$$

(b) The number of orbits $=\dfrac{5.0\times10^9\text{ years}}{2.05\times10^8\text{ years}}=24$ orbits.

(c) Applying Newton's second law yields

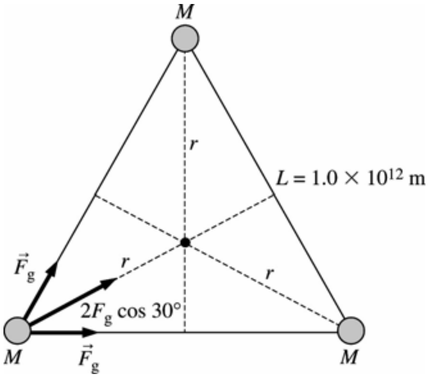
$$\frac{GM_{\text{g center}}m_{\text{ss}}}{r^2}=\frac{m_{\text{ss}}v^2}{r}\quad\Rightarrow\quad M_{\text{g center}}=\frac{v^2r}{G}=\frac{(2.30\times10^5\text{ m/s})^2(2.36\times10^{20}\text{ m})}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}=1.9\times10^{41}\text{ kg}$$

(d) The number of stars in the center of the galaxy is

$$\frac{1.87\times10^{41}\text{ kg}}{1.99\times10^{30}\text{ kg}}=9.4\times10^{10}$$

13.61. Model: Assume the three stars are spherical masses.

Visualize:



The stars rotate about the center of mass, which is the center of the triangle and equal distance *r* from all three stars. The gravitational force between any two stars is the same. On a given star the two forces from the other stars make an angle of 60°.

Solve: The value of *r* can be found as follows:

$$\frac{L/2}{r} = \cos(30°) \quad \Rightarrow \quad r = \frac{L}{2\cos(30°)} = \frac{1.0\times10^{12}\ \text{m}}{2\cos(30°)} = 0.577\times10^{12}\ \text{m}$$

The gravitational force between any two stars is

$$F_g = \frac{GM^2}{L^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})^2}{(1.0\times10^{12}\ \text{m})^2} = 2.64\times10^{26}\ \text{N}$$

The component of this force toward the center is

$$F_c = F_g\cos(30°) = (2.64\times10^{26}\ \text{N})\cos(30°) = 2.29\times10^{26}\ \text{N}$$

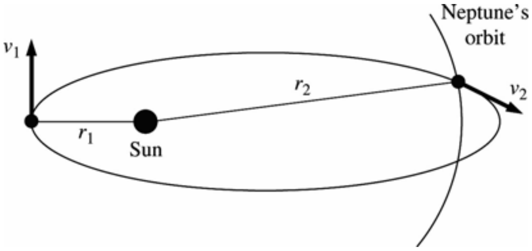
The net force on a star toward the center is twice this force, and that force equals $MR\omega^2$. This means

$$2\times2.29\times10^{26}\ \text{N} = MR\omega^2 = MR\left(\frac{2\pi}{T}\right)^2$$

$$T = \sqrt{\frac{4\pi^2MR}{4.58\times10^{26}\ \text{N}}} = \sqrt{\frac{4\pi^2(1.99\times10^{30}\ \text{kg})(0.577\times10^{12}\ \text{m})}{4.58\times10^{26}\ \text{N}}} = 3.15\times10^8\ \text{s} = 10\ \text{years}$$

13.62. Model: For the sun + comet system, the mechanical energy is conserved.

Visualize:



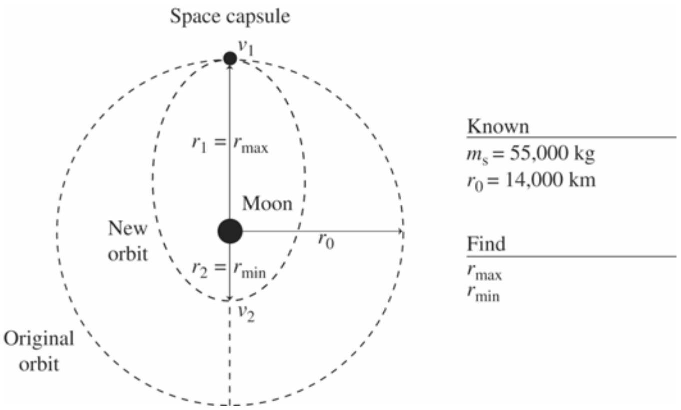
Solve: The conservation of energy equation $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i}$ gives

$$\frac{1}{2}M_\text{c}v_2^2 - \frac{GM_\text{s}M_\text{c}}{r_2} = \frac{1}{2}M_\text{c}v_1^2 - \frac{GM_\text{s}M_\text{c}}{r_1}$$

Using $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$, $M_\text{s} = 1.99 \times 10^{30}$ kg, $r_1 = 8.79 \times 10^{10}$ m, $r_2 = 4.50 \times 10^{12}$ m, and $v_1 = 54.6$ km/s, we get $v_2 = 4.49$ km/s.

13.63. Model: Model the planet (p) as a spherical mass and the spaceship (s) as a point mass. Also assume the Δt for the rocket firing is negligibly small.

Visualize:



Solve: (a) As we see from the diagram, The spaceship’s maximum distance is $r_\text{max} = r_0 = 14{,}000$ km.

(b) For the circular motion of the spaceship around the planet,

$$\frac{GM_\text{p}m_\text{s}}{r_0^2} = \frac{mv_0^2}{r_0} \quad \Rightarrow \quad v_0 = \sqrt{\frac{GM_\text{p}}{r_0}}$$

Immediately after the rockets were fired $v_1 = v_0/2$ and $r_1 = r_0$. Therefore,

$$v_1 = \frac{1}{2}\sqrt{\frac{GM_\text{p}}{r_0}}$$

Its minimum distance r_2 occurs at the other end of the ellipse. The energy at the firing point is equal to the energy at the other end of the elliptical trajectory. That is,

$$\frac{1}{2}m_\text{s}v_1^2 - \frac{GM_\text{p}m_\text{s}}{r_1} = \frac{1}{2}m_\text{s}v_2^2 - \frac{GM_\text{p}m_\text{s}}{r_2}$$

Since the angular momentum is conserved (the same at r_min and r_max), we have

$$mv_1r_1 = mv_2r_2 \quad \Rightarrow \quad v_2 = v_1(r_1/r_2)$$

With this expression for v_2, the energy equation simplifies to

$$\frac{1}{2}v_1^2 - \frac{GM_\text{p}}{r_1} = \frac{1}{2}v_1^2(r_1/r_2)^2 - \frac{GM_\text{p}}{r_2}$$

Using $r_1 = r_0$ and $v_1 = v_0/2 = \frac{1}{2}\sqrt{\frac{GM_\text{p}}{r_0}}$,

$$\frac{1}{2}\left(\frac{1}{4}\frac{GM_\text{p}}{r_0}\right) - \frac{GM_\text{p}}{r_0} = \frac{1}{2}\left(\frac{1}{4}\frac{GM_\text{p}}{r_0}\right)\frac{r_0^2}{r_2^2} - \frac{GM_\text{p}}{r_2} \quad \Rightarrow \quad \frac{1}{8r_0} - \frac{1}{r_0} = \frac{r_0}{8r_2^2} - \frac{1}{r_2}$$

$$\frac{7}{8r_0} + \frac{r_0}{8r_2^2} - \frac{1}{r_2} = 0 \quad \Rightarrow \quad \left(\frac{7}{8r_0}\right)r_2^2 - r_2 + \frac{r_0}{8} = 0$$

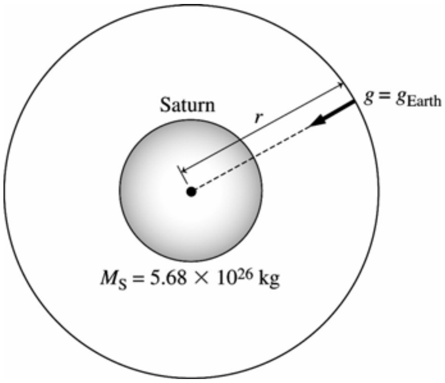
The solutions are $r_2 = r_0$ (the initial distance) and $r_2 = r_0/7$. Thus, the minimum distance is

$$r_\text{min} = \frac{r_0}{7} + \frac{14{,}000 \text{ km}}{7} = 2000 \text{ km}$$

Assess: We did not need to know the mass of the space capsule.

13.64. Solve: (a) At what distance from the center of Saturn is the acceleration due to gravity the same as on the surface of the earth?

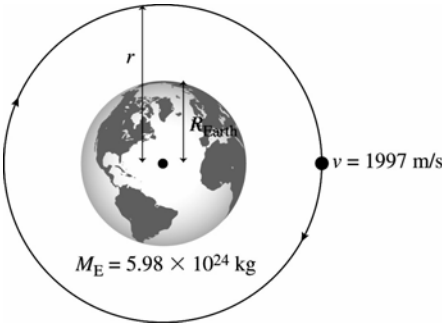
(b)



(c) The distance is 6.21×10^7 m. This is $1.06R_\text{Saturn}$

13.65. Solve: (a) A 1000 kg satellite orbits the earth with a speed of 1997 m/s. What is the radius of the orbit?

(b)

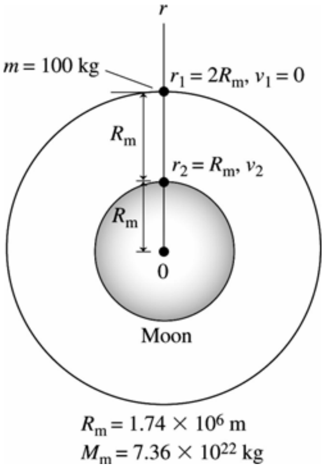


(c) The radius of the orbit is

$$r = \frac{GM_\text{E}}{(v_\text{payload})^2} = \frac{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24} \text{ kg})}{(1997 \text{ m/s})^2} = 1.00\times10^8 \text{ m}$$

13.66. Solve: (a) A 100 kg object is released from rest at an altitude above the moon equal to the moon's radius. At what speed does it impact the moon's surface?

(b)

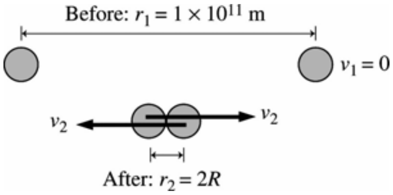


(c) The speed is $v_2 = 1680$ m/s.

Challenge Problems

13.67. Model: Gravity is a conservative force, so we can use conservation of energy.

Visualize:



The planets will be pulled together by gravity and each will have speed v_2 as they crash and the separation between their centers will be $2R$.

Solve: The planets begin with only gravitational potential energy. When they crash, they have both potential and kinetic energy. Thus,

$$K_2 + U_2 = \frac{1}{2}Mv_2^2 + \frac{1}{2}Mv_2^2 - \frac{GMM}{r_2} = K_1 + U_1 = 0\text{ J} - \frac{GMM}{r_1}$$

$$v_2 = \sqrt{GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

Because the planet is "Jupiter-size," we'll use $M = M_{\text{Jupiter}} = 1.9 \times 10^{27}$ kg and $r_2 = 2R_{\text{Jupiter}} = 1.4 \times 10^8$ m. Inserting these values into the expression above gives the crash speed of each planet as $v_2 = 3.0 \times 10^4$ m/s.

Assess: Note that the force is not constant, because it varies with distance, so the motion is *not* constant acceleration motion. The formulas from constant-acceleration kinematics do not work for problems such as this.

13.68. Solve: (a) Kepler's third law for circular orbits is

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3 \quad \Rightarrow \quad T = \sqrt{\frac{4\pi^2}{GM}}r^{\frac{3}{2}}$$

Letting $a=\sqrt{\dfrac{4\pi^2}{GM}}$, the first satellite obeys $T=ar^{\frac{3}{2}}$. For the second satellite, which orbits the same mass,

$$T+\Delta T=a(r+\Delta r)^{\frac{3}{2}}=ar^{\frac{3}{2}}\left(1+\frac{\Delta r}{r}\right)^{\frac{3}{2}}$$

Since $\dfrac{\Delta r}{r}\ll 1$, we can use the approximation $(1\pm x)^n\approx 1\pm nx$.

Thus

$$T+\Delta T\approx ar^{\frac{3}{2}}\left(1+\frac{3}{2}\frac{\Delta r}{r}\right)$$

Subtracting the equation $T=ar^{\frac{3}{2}}$ for the first satellite from this,

$$\Delta T=\frac{3}{2}ar^{\frac{3}{2}}\left(\frac{\Delta r}{r}\right)$$

Dividing this by the equation for the first satellite,

$$\frac{\Delta T}{T}=\frac{3}{2}\frac{\Delta r}{r}$$

(b) The satellites orbit the earth. The fractional difference in their periods is

$$\frac{\Delta T}{T}=\frac{3}{2}\frac{1\text{ km}}{6700\text{ km}}=2.24\times10^{-4}$$

After $\dfrac{1}{2.24\times10^{-4}}=4467$ periods they will meet again. For the inner satellite,

$$\begin{aligned}T&=\sqrt{\frac{4\pi^2}{G(5.98\times10^{24}\text{ kg})}}(6.700\times10^6\text{ m})^{\frac{3}{2}}\\&=5456\text{ s}=1.52\text{ hrs}\end{aligned}$$

So the satellites will meet again in $4467\times1.52\text{ hrs}=6770\text{ hrs}=282\text{ days}$.

Assess: A communications satellite has an orbital period of around 1.5 h. The surprising length of time between the two satellites meeting is due to the small differences in their periods.

13.69. Model: Planet Physics is a spherical mass. The cruise ship is in a circular orbit.

Solve: (a) At the surface, the free-fall acceleration is $g=GM/R^2$. From kinematics,

$$\begin{aligned}&y_f=y_i+v_0\Delta t-2g(\Delta t)^2\quad\Rightarrow\quad 0\text{ m}=0\text{ m}+(11\text{ m/s})(2.5\text{ s})-2g(2.5\text{ s})^2\\&g=2.20\text{ m/s}^2\end{aligned}$$

The period of the cruise ship's orbit is $230\times60=13{,}800$ s. For the circular orbit of the cruise ship,

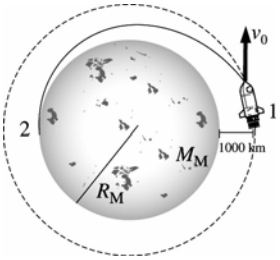
$$T^2=\left(\frac{4\pi^2}{GM}\right)(2R)^3\quad\Rightarrow\quad\frac{T^2}{32\pi^2}=R\left(\frac{R^2}{GM}\right)=R\left(\frac{1}{g}\right)$$

$$R=\frac{(2.20\text{ m/s}^2)(13{,}800\text{ s})^2}{32\pi^2}=1.327\times10^6\text{ m.}$$

The mass is thus $M=(R^2/G)g=5.8\times10^{22}$ kg.

(b) From part (a), $R=1.3\times10^6$ m.

13.70. Model: The moon is a spherical mass. The moon lander is originally in a circular orbit.
Visualize:



Solve: Energy and momentum are conserved between points 1 and 2 in the elliptical orbit. Also, at both points $\beta = 90°$, so $L = mrv\sin\beta = mrv$. Let $h = 1000$ km. The original speed of the lander is $v_0 = \sqrt{\frac{GM_m}{R_m + h}} = 1338$ m/s. Conservation of angular momentum requires $mr_1v_1 = mr_2v_2 \Rightarrow \frac{v_2}{v_1} = \frac{r_1}{r_2} = \frac{R_m + h}{R_m}$. The energy conservation equation gives

$$\frac{1}{2}mv_1{}^2 - G\frac{M_m m}{R_m + h} = \frac{1}{2}mv_2^2 - G\frac{M_m m}{R_m}$$

Substituting $v_2 = \left(\frac{R_m + h}{R_m}\right)v_1$,

$$\frac{1}{2}v_1^2 - G\frac{M_m}{R_m + h} = \frac{1}{2}v_1^2\left(\frac{R_m + h}{R_m}\right)^2 - G\frac{M_m}{R_m}$$

$$v_1^2\left[1 - \left(\frac{R_m + h}{R_m}\right)^2\right] = 2GM_m\left(\frac{1}{R_m + h} - \frac{1}{R_m}\right)$$

$$v_1^2 = \frac{2GM_m h}{R_m(R_m + h)}\frac{1}{[(R_m + h)/R_m]^2 - 1} = \frac{2GM_m}{(R_m + h)}\left(\frac{R_m}{2R_m + h}\right) = 1.392 \times 10^6 \text{ m}^2/\text{s}^2$$

$$v_1 = 1180 \text{ m/s}$$

The fractional change in speed required to just graze the moon at point 2 is

$$\frac{1338 \text{ m/s} - 1180 \text{ m/s}}{1338 \text{ m/s}} = 11.8\%$$

Assess: A reduction in speed by almost 12% is reasonable.

13.71. Model: Model the earth as a spherical mass and the satellite as a point mass. This is an isolated system, so mechanical energy is conserved. Also, the angular momentum of the satellite is conserved.
Visualize: Please refer to Figure CP13.71.
Solve: (a) Angular momentum is $L = mrv\sin\beta$. The angle $\beta = 90°$ at points 1 and 2, so conservation of angular momentum requires

$$mr_1v_1' = mr_2v_2' \quad \Rightarrow \quad v_1' = \left(\frac{r_2}{r_1}\right)v_2'$$

The energy conservation equation is

$$\frac{1}{2}m(v_2')^2 - \frac{GMm}{r_2} = \frac{1}{2}m(v_1')^2 - \frac{GMm}{r_1} \quad \Rightarrow \quad (v_2')^2 - (v_1')^2 = 2GM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Using the angular momentum result for v_1' gives

$$(v_2')^2-(v_2')^2\left(\frac{r_2}{r_1}\right)^2=2GM\left(\frac{r_1-r_2}{r_1r_2}\right)\ \Rightarrow\ (v_2')^2\left[\frac{r_1^2-r_2^2}{r_1^2}\right]=2GM\left(\frac{r_1-r_2}{r_1r_2}\right)$$

$$v_2'=\sqrt{\frac{2GM(r_1/r_2)}{r_1+r_2}}\quad\text{and}\quad v_1'=\left(\frac{r_2}{r_1}\right)v_2'=\sqrt{\frac{2GM(r_1/r_2)}{r_1+r_2}}$$

(b) For the circular orbit,

$$v_1=\sqrt{\frac{GM}{r_1}}=\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg})}{(6.37\times10^6\ \text{m}+3\times10^5\ \text{m})}}=7730\ \text{m/s}$$

For the elliptical orbit,

$$r_1=R_\text{e}+300\ \text{km}=6.37\times10^6\ \text{m}+3\times10^5\ \text{m}=6.67\times10^6\ \text{m}$$

$$r_2=R_\text{e}+35{,}900\ \text{km}=6.37\times10^6\ \text{m}+3.59\times10^7\ \text{m}=4.23\times10^7\ \text{m}$$

$$v_1'=\sqrt{\frac{2GM(r_1/r_2)}{r_1+r_2}}\ \Rightarrow\ v_1'=10{,}160\ \text{m/s}$$

(c) From the work-kinetic energy theorem, $W=\Delta K=\frac{1}{2}mv_1'^2-\frac{1}{2}mv_1^2=2.17\times10^{10}\ \text{J}$

(d) $v_2'=\sqrt{\frac{2GM(r_1/r_2)}{r_1+r_2}}$

Using the same values of r_1 and r_2 as in (b), $v_2'=1600$ m/s. For the circular orbit,

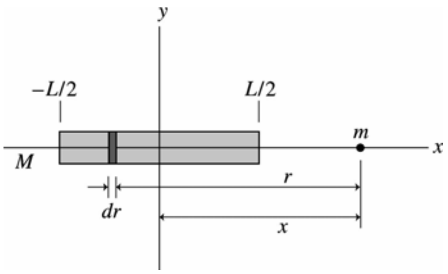
$$v_2=\sqrt{\frac{GM}{r_2}}=3070\ \text{m/s}$$

(e) $W=\frac{1}{2}mv_2^2-\frac{1}{2}mv_2'^2=3.43\times10^9\ \text{J}$

(f) The total work done is 2.52×10^{10} J. This is the same as in Example 13.6, but here we've learned how the work has to be divided between the two burns.

13.72. Model: The rod is thin and uniform.

Visualize:



Solve: (a) The rod is not spherical and so must be divided into thin sections each *dr* wide and having mass *dm*. Since the rod is uniform,

$$\frac{dm}{M}=\frac{dr}{L}\ \Rightarrow\ dm=\frac{M}{L}dr$$

The width of the rod is small enough so that all of *dm* is distance *r* away from *m*. The gravitational potential energy of *dm* and *m* is

$$dU = -G\frac{m\,dm}{r} = -G\frac{m(M/L)dr}{r}$$

The total potential energy of the rod and mass *m* is found by adding the contributions *dU* from every point along the rod in an integral:

$$U = \int dU = -\frac{GMm}{L}\int_{x-\frac{L}{2}}^{x+\frac{L}{2}}\frac{dr}{r} = -\frac{GMm}{L}\ln\left(\frac{x+L/2}{x-L/2}\right)$$

Note *r* is increasing with the limits chosen as they are.

(b) The force on *m* when at *x* is

$$F = -\frac{dU}{dx} = \frac{GMm}{L}\frac{d}{dx}\left[\ln\left(x+\frac{L}{2}\right) - \ln\left(x-\frac{L}{2}\right)\right] = \frac{GMm}{L}\left(\frac{1}{x+L/2} - \frac{1}{x-L/2}\right)$$

$$= -GMm\left(\frac{4}{4x^2 - L^2}\right), \quad x \ge \frac{L}{2}$$

Assess: The direction of the force is toward the $-x$ direction, as expected. The force magnitude approaches ∞ as the mass *m* approaches the end of the rod, but goes to zero like x^{-2} as *x* gets large. This is expected since from far away the rod looks like a point mass.